The following questions are worth 30 points total, and will be added to your WebAssign scores. Only correct answers will be accepted. Due 11/14/2008.

The tangent line approximation \( L(x) \) is the best linear approximation to \( f(x) \) near \( x = a \) because \( f(x) \) and \( L(x) \) have the same rate of change (derivative) at \( a \). For a better approximation than a linear one, let’s try a second-degree (quadratic) approximation \( P_2(x) \). In other words, we approximate a curve by a parabola instead of by a straight line. To make sure the approximation is a good one, we stipulate the following:

\[
\begin{align*}
P_2(a) &= f(a) \\
P'_2(a) &= f'(a) \\
P''_2(a) &= f''(a)
\end{align*}
\]

1. Find the quadratic approximation \( P_2(x) = Ax + Bx + Cx^2 \) to the function \( f(x) = e^x \) that satisfies the above three conditions with \( a = 0 \). Graph \( P_2 \) and \( f \) on the same axis. Does \( P_2 \) fit \( f \) better than a tangent line in the region where \( a = 0 \)?

2. If we repeat this method for higher and higher-degree polynomials, we find that \( f(x) \) can be better approximated. Now repeat this method until you get a forth degree polynomial, \( P_4(x) = Ax + Bx + Cx^2 + Dx^3 + Ex^4 \), that approximates \( f(x) \) when \( a = 0 \). As before, graph \( P_4 \) and \( f \) on the same axis. Does the forth degree \( P_4 \) fit \( f \) better second degree \( P_2 \) in the region where \( a = 0 \)?

3. Without repeating this method, can you predict the tenth degree polynomial approximation to \( e^x \)?

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1This document was prepared by Ron Bannon using \LaTeX. Source and pdf are available by emailing a request to rbannon@mac.com.

2We did this in class!