Name: 
Signature: 

Show all work clearly and in order, and box your final answers. Justify your answers whenever possible. You have 20 minutes to take this 10 point quiz.

1. 5 points Show that

\[ y = f(x) = A \sinh(mx) + B \cosh(mx) \]

satisfies the differential equation

\[ y'' = m^2y. \]

**Solution:** You’ll need to find the first and second derivative.

\[
\begin{align*}
y &= A \sinh(mx) + B \cosh(mx) \\
y' &= Am \cosh(mx) + Bm \sinh(mx) \\
y'' &= Am^2 \sinh(mx) + Bm^2 \cosh(mx)
\end{align*}
\]

Then plug it in! And then simplify the right side to show that it is the same as the left side.

\[
\begin{align*}
y'' &= m^2y \\
Am^2 \sinh(mx) + Bm^2 \cosh(mx) &= m^2(A \sinh(mx) + B \cosh(mx)) \\
Am^2 \sinh(mx) + Bm^2 \cosh(mx) &= Am^2 \sinh(mx) + Bm^2 \cosh(mx)
\end{align*}
\]

Yes, the left and right sides are identical.

2. 5 points Using the above problem, Find \( y = f(x) \), such that \( y'' = 9y \), \( y = f(0) = -4 \), and \( y' = f'(0) = 6 \).

**Solution:** Well, we’re told that \( y'' = m^2y \) and that \( y'' = 9y \), so \( m \) could either be 3 or \(-3\). Now let’s look at \( y = f(0) = -4 \), here we have

\[
\begin{align*}
f(x) &= A \sinh(mx) + B \cosh(mx) \\
f(0) &= A \sinh 0 + B \cosh 0 \\
-4 &= B.
\end{align*}
\]

Now let’s look at \( y' = f'(0) = 6 \), here we have

\[
\begin{align*}
f'(x) &= Am \cosh(mx) + Bm \sinh(mx) \\
f'(0) &= Am \cosh 0 + Bm \sinh 0 \\
6 &= Am.
\end{align*}
\]
Okay, the only tricky part is that we have two possible values of $m$. Let’s see what happens if $m = 3$?

$$y = f(x) = 2 \sinh 3x - 4 \cosh 3x$$

And now what if $m = -3$.

$$y = f(x) = -2 \sinh (-3x) - 4 \cosh (-3x)$$

Recalling the even/odd properties, this equation actually becomes

$$y = f(x) = 2 \sinh 3x - 4 \cosh 3x$$

So there’s really only one choice.

$$y = f(x) = 2 \sinh 3x - 4 \cosh 3x$$