1. Given that 

\[ f(x) = xe^x. \]

Here you are given both the first and second derivative to be used to analyze the supplied graph of \( f \).

\[ f'(x) = (x + 1)e^x \quad \text{and} \quad f''(x) = (x + 2)e^x \]

Figure 1: Partial Graph of \( f \)

Answer each of the following questions.

(a) [1 point] Find \( f \)'s domain.

Solution: The domain is \([\mathbb{R}]\).

(b) [1 point] Find \( x \)- and \( y \)-intercepts.

Solution: The \( x \)- and \( y \)-intercepts are both \([0, 0]\).

(c) [1 point] Is \( f \) even, odd, neither?

Solution: By graphical inspection, or even if you use the tests, you should know that this function is not \textit{even} or \textit{odd}. So \( f \) is \textit{neither}.
(d) 1 point Find where $f(x)$ is increasing.

**Solution:** Using $f'$ and simple sign analysis and you will find that $f(x)$ is increasing on

$$(-1, \infty).$$

This is consistent with the graph of $f(x)$.

(e) 1 point Find where $f(x)$ is decreasing.

**Solution:** Using $f'$ and simple sign analysis and you will find that $f(x)$ is decreasing on

$$(-\infty, -1).$$

This is consistent with the graph of $f(x)$.

(f) 1 point Find the local/global minimum point.

**Solution:** From above, the local and global minimum is:

$$(-1, -e^{-1})$$

(g) 2 points Find the inflection point.\(^1\)

**Solution:** Using the second derivative you should see that the concavity changes at:

$$(-2, -2e^{-2})$$

(h) 2 points What’s $f$’s range?

**Solution:** From the information above, we have

$$[-e^{-1}, \infty)$$

\(^1\)The point on $f$ where the second derivative changes sign.