Name: ______________________________

Signature: ______________________________

This is a take home exam (honor system) and is due at 8:30 AM on November 26, 2008. **Do not work together!**

1. Given

$$f(x) = \sqrt[3]{x}e^{-x^2},$$

and a partial graph of f.

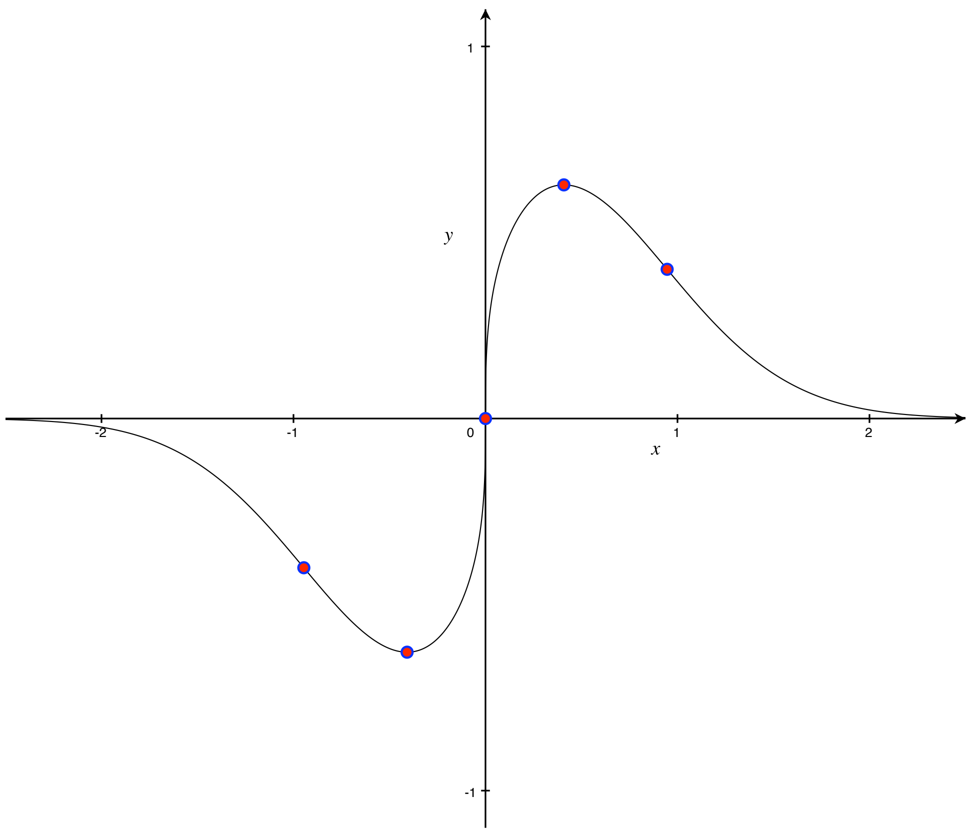

Figure 1: Partial graph of f with some important points indicated.

Questions about f are on the next page.

Answer each of the following questions. I don't need to see your work. It's okay to use a computer, but do not use *decimals*—exact answers only! Answers must be $\boxed{\text{BOXED}}$.

(a) $\boxed{\text{3 points}}$ Find and simplify f'.

Solution:

$$\boxed{f'(x) = \frac{1-6x^2}{3e^{x^2}\sqrt[3]{x^2}}}$$

(b) $\boxed{\text{4 points}}$ Find and simplify f''.

Solution:

$$\boxed{f''(x) = \frac{2\left(18x^4-15x^2-1\right)}{9xe^{x^2}\sqrt[3]{x^2}}}$$

(c) $\boxed{\text{3 points}}$ Find the maximal point on f.

Solution: Certainly the graph helps and it should be clear that a maximum (both global and local) occurs at:

$$\boxed{\left(\frac{1}{\sqrt{6}},\ f\left(\frac{1}{\sqrt{6}}\right)\right) \quad \text{or} \quad \left(\frac{1}{\sqrt{6}},\ \frac{1}{\sqrt[6]{6e}}\right)}$$

Approximating this point may prove helpful.

$$(0.41,\ 0.63)$$

(d) $\boxed{\text{3 points}}$ Find the minimal point on f.

Solution: Certainly the graph helps and it should be clear that a minimum (both global and local) occurs at:

$$\boxed{\left(\frac{1}{\sqrt{6}},\ f\left(\frac{1}{\sqrt{6}}\right)\right) \quad \text{or} \quad \left(\frac{1}{\sqrt{6}},\ \frac{1}{\sqrt[6]{6e}}\right)}$$

Approximating this point may prove helpful.

$$(-0.41,\ -0.63)$$

(e) 9 points Find the points-of-inflection on f.

Solution: Certainly the graph helps and it should be clear that a there are three points-of-inflection and they occur at the following x values:

$$-\sqrt{\frac{5+\sqrt{3}}{12}},\ 0,\sqrt{\frac{5+\sqrt{3}}{12}}$$

The points are:

$$\boxed{\left(-\sqrt{\frac{5+\sqrt{3}}{12}},\ f\left(-\sqrt{\frac{5+\sqrt{3}}{12}}\right)\right),\ (0,\ 0),\ \left(\sqrt{\frac{5+\sqrt{3}}{12}},\ f\left(\sqrt{\frac{5+\sqrt{3}}{12}}\right)\right)}$$

Approximating these points may prove helpful.

$$(-0.95,\ -0.40),\ (0.00,\ 0.00),\ (0.95,\ 0.40)$$