1. If \( f(x) = \sin\left(x^2 e^{x^2}\right) \), find \( f'(x) \).

**Solution:**

\[
f'(x) = \cos\left(x^2 e^{x^2}\right) \cdot \left(2xe^{x^2} + x^2 e^{x^2} \cdot 2x\right)
\]

2. If \( f(t) = \frac{18}{3 + t^2} \), find \( f'(t) \).

**Solution:**

\[
f'(t) = -\frac{36t}{(3 + t^2)^2}
\]

3. If a snowball melts so that its surface area decreases at a rate of 4 cm\(^2\)/min, find the rate at which the diameter decreases when the diameter is 39 cm.

**Solution:**

\[
S = \pi d^2 \quad \Rightarrow \quad \frac{dS}{dt} = 2\pi d \frac{dd}{dt} \quad \Rightarrow \quad -4 = 78\pi \left. \frac{dd}{dt} \right|_{d=39}
\]

Solving for the diameters rate,

\[
\left. \frac{dd}{dt} \right|_{d=39} = -\frac{2}{39\pi} \approx -0.016 \text{ cm/min}
\]

4. If \( f(x) = x (\ln x)^{-1} \), find \( f'(e^3) \).

**Solution:**

\[
f(x) = x (\ln x)^{-1} \quad \Rightarrow \quad f'(x) = \frac{\ln x - 1}{(\ln x)^2} \quad \Rightarrow \quad f'(e^3) = \frac{2}{9}
\]
5. The position \((s\text{ in meters}, \text{ and } t\text{ in seconds})\) function of a particle is given by \(s = t^3 - 3t^2 - 5t\), for \(t \geq 0\). When does the particle reach a velocity of 139 meters per second?

**Solution:**

\[
s = t^3 - 3t^2 - 5t \\
\frac{ds}{dt} = v(t) = 3t^2 - 6t - 5 \\
139 = 3t^2 - 6t - 5 \\
0 = 3t^2 - 6t - 144 \\
0 = 3(t + 6)(t - 8)
\]

So the particle reach a velocity of 139 meters per second at \(t = 8\text{ seconds}\).

6. If \(f(x) = \ln \left| \frac{x^2 - 4}{x^2 + 1} \right|\).

**Solution:** I admit that this is a tricky problem, but I want to note that if \(f(x) = \ln |x|\), then \(f'(x) = 1/x\). I will show this is true if anyone is interested, just ask!

\[
f'(x) = \frac{10x}{(x^2 - 4)(x^2 + 1)} = \frac{10x}{x^4 - 3x^2 + 1}
\]

7. A machinist is required to manufacture a circular metal disk with area 1,000 cm\(^2\).
   
   (a) What radius (exact and approximate to three decimal places) produces such a disk?

**Solution:**

\[
\pi r^2 = 1000 \text{ cm}^2 \quad \Rightarrow \quad r = \sqrt{\frac{1000}{\pi}} \text{ cm} \approx 17.841 \text{ cm}
\]

(b) If the machinist is allowed an error tolerance of \(\pm 5\) cm\(^2\) in the area of the disk, how close to the ideal radius in part(a) must the machinist control the radius (three decimal places)?

**Solution:**

\[
\sqrt{\frac{1000}{\pi}} - \sqrt{\frac{995}{\pi}} \approx 0.044658996605 \\
\sqrt{\frac{1005}{\pi}} - \sqrt{\frac{1000}{\pi}} \approx 0.044547487976
\]

So the machinist is allowed \(0.045\text{ cm}\).
8. If \( f(3) = 4, \ g(3) = 2, \ f'(3) = -5, \) and \( g'(3) = 6, \) find each of the following:

(a) \((f + g)'(3)\)

Solution:
\[
(f + g)'(3) = f'(3) + g'(3) = -5 + 6 = 1
\]

(b) \((f \cdot g)'(3)\)

Solution:
\[
(f \cdot g)'(3) = f(3) \cdot g'(3) + f'(3) \cdot g(3) = 4 \cdot 6 - 5 \cdot 2 = 14
\]

(c) \(\left(\frac{f}{g}\right)'(3)\)

Solution:
\[
\left(\frac{f}{g}\right)'(3) = \frac{g(3) \cdot f'(3) - g'(3) \cdot f(3)}{[g(3)]^2} = \frac{2 \cdot (-5) - 6 \cdot 4}{[2]^2} = \frac{-17}{2}
\]

(d) \(\left(\frac{f}{f - g}\right)'(3)\)

Solution:
\[
\left(\frac{f}{f - g}\right)'(3) = \frac{\left[f(3) - g(3)\right] \cdot f'(3) - [f'(3) - g'(3)] \cdot f(3)}{[f(3) - g(3)]^2}
= \frac{[4 - 2] \cdot (-5) - [-5 - 6] \cdot 4}{[4 - 2]^2} = \frac{17}{2}
\]

9. Use the definition of the derivative to find \( f'(-2), \) where \( f(x) = x^3 - 2x. \)

Solution:
\[
f'(-2) = \lim_{h \to 0} \frac{f(-2 + h) - f(-2)}{h}
= \lim_{h \to 0} \frac{(h^3 - 6h^2 + 10h - 4) + 4}{h}
= \lim_{h \to 0} \frac{h^3 - 6h^2 + 10h}{h}
= \lim_{h \to 0} h^2 - 6h + 10 = 10
\]
10. Find an equation of the tangent line to curve $y = x^3 - 2x$ at the point $(2, 4)$.

**Solution:** $f'(x) = 3x^2 - 2$, so $f'(2) = 10$.

$$y - 4 = 10(x - 2)$$

11. **Optional Problem:** Use a graph to find $\delta$ a number such that $|\sqrt{4x+1} - 3| < 0.6$ whenever $|x - 2| < \delta$.

**Solution:** Using the graph below the $\delta \leq 0.81$.

![Figure 1: Partial Graph of $f(x) = |\sqrt{4x+1} - 3|$, $y = 0.6$, $x = 1.19$, and $x = 2.99$.](image)

12. If $g(x) = \sqrt{3 - 5x}$, find the domain of $g'(x)$.

**Solution:** The domain of

$$g'(x) = -\frac{5}{2\sqrt{3 - 5x}}$$

is $(-\infty, 3/5]$.
13. Evaluate the limit.

\[
\lim_{x \to 0} \frac{(2 + x)^{-1} - 2^{-1}}{x}
\]

Solution:

\[
\lim_{x \to 0} \frac{(2 + x)^{-1} - 2^{-1}}{x} = \lim_{x \to 0} \frac{(2 + x)^{-1} - 2^{-1}}{x} \cdot \frac{2 (2 + x)}{2 (2 + x)} = \lim_{x \to 0} \frac{2 - (2 + x)}{2x (2 + x)} = \lim_{x \to 0} \frac{-x}{2x (2 + x)} = \lim_{x \to 0} \frac{-1}{2 (2 + x)} = \frac{-1}{4}
\]

14. Find the derivative of the function.

\[f(x) = 14 - 3x + 5x^2\]

Solution:

\[f'(x) = -3 + 10x\]

15. If a cylindrical tank holds 100,000 gallons of water, which can be drained from the bottom of the tank in an hour, then Torricelli’s Law gives the volume of water remaining in the tank after \(t\) minutes as

\[V(t) = 100000 \left(1 - \frac{t}{65}\right)^2, \quad 0 \leq t \leq 60.\]

Find the rate at which the water is flowing out of the tank (the instantaneous rate of change of \(V\) with respect to \(t\)) as a function of \(t\).

Solution:

\[V'(t) = -\frac{200000}{65} \left(1 - \frac{t}{65}\right)\]
16. Determine where \( f \) is discontinuous.

\[
f(x) = \begin{cases} 
\sqrt{-x} & \text{if } x < 0 \\
3 - x & \text{if } 0 \leq x < 3 \\
(3 - x)^2 & \text{if } x > 3 
\end{cases}
\]

**Solution:** Unfortunately, the graph may be misleading because there only appears to be one discontinuity at \( x = 0 \). Using the definition of continuity, we should also know that \( f \) is discontinuous at \( x = 3 \). So \( f \) is discontinuous at \( x = 0 \) and \( x = 3 \).

Here’s the graph.

![Partial Graph of \( f(x) \).](image)

17. If

\[
f(t) = \frac{13}{3 + t^2}
\]

find \( f'(t) \).

**Solution:**

\[
f'(t) = \frac{-26t}{(3 + t^2)^2}
\]
18. At what point is the function \( f(x) = |6 - x| \) not differentiable.

**Solution:** At \( t = 6 \).

19. For \( x = 5 \), determine whether is continuous from the right, from the left, or neither.

![Partial Graph](image)

**Figure 3: Partial Graph**

**Solution:** It is continuous from both the right and left at \( x = 5 \).

20. Differentiate.

\[
g(x) = x^7 \cos x
\]

**Solution:**

\[
g'(x) = 7x^6 \cos x - x^7 \sin x
\]

21. Find the points on the curve \( y = 2x^3 + 3x^2 - 12x + 1 \) where the tangent is horizontal.

**Solution:** The derivative is

\[
y' = 6x^2 + 6x - 12 = 6(x - 1)(x + 2),
\]
and is equal to zero (horizontal tangent) at \( x = 1 \) and \( x = -2 \), so the points are:

\[
(1, -6) \quad \text{and} \quad (-2, 21)
\]
22. **Optional Problem:** Use a graph to find a whole number $N$ such that

$$\left| \frac{6x^2 + 5x - 3}{2x^2 - 1} - 3 \right| < 0.3$$

whenever $x > N$.

**Solution:** Certainly we see that the value for $x$ is somewhere between 8 and 9, so the answer is $N \geq 9$.

![Graph](image)

Figure 4: Partial Graph of $f(x) = \left| \frac{6x^2 + 5x - 3}{2x^2 - 1} - 3 \right|$ and $y = 0.3$.

23. Find the equation of the tangent to the curve at the given point.

$$y = \sqrt{1 + 4 \sin x}, \quad (0, 1)$$

**Solution:** The derivative is

$$y' = \frac{2 \cos x}{\sqrt{1 + 4 \sin x}},$$

and the slope of the tangent line at the point $(0, 1)$ is

$$y'(0) = \frac{2 \cos (0)}{\sqrt{1 + 4 \sin (0)}} = 2.$$

So the equation of the tangent to the curve, $y = \sqrt{1 + 4 \sin x}$, at the point $(0, 1)$ is

$$y - 1 = 2x$$
24. Find the limit.

\[ \lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos 2x} \]

Solution:

\[
\lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos 2x} = \lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos^2 x - \sin^2 x} = \lim_{x \to \frac{\pi}{4}} (\sin x - \cos x) (\cos x + \sin x) = \lim_{x \to \frac{\pi}{4}} \frac{-1}{\cos x + \sin x} = -\frac{\sqrt{2}}{2}
\]

25. Calculate \( y' \).

\[ y = \sqrt{x} \cos \sqrt{x} \]

Solution:

\[
y' = \frac{\cos \sqrt{x} - \sqrt{x} \sin \sqrt{x}}{2\sqrt{x}}
\]

26. A spherical balloon is being inflated. Find the rate of increase of the surface area with respect to the radius \( r \) when \( r = 1 \) foot.

Solution: The formula (it is in your book) for a sphere’s surface area is \( A = 4\pi r^2 \). Differentiating with respect to \( r \)

\[
\frac{d}{dr} (A) = \frac{d}{dr} (4\pi r^2) \Rightarrow \frac{dA}{dr} = 8\pi r.
\]

Here, we need to evaluate this derivative at \( r = 1 \) foot

\[ \frac{dA}{dr} \bigg|_{r=1} = 8\pi \text{ feet} \]
27. Differentiate the function.

\[ f(x) = \frac{\sqrt{7}}{x^3} \]

**Solution:**

\[ f'(x) = -\frac{5\sqrt{7}}{x^6} \]

28. Find an equation of the tangent line to the curve.

\[ y = \frac{\sqrt{x}}{x + 6}, \text{ at the point } P(4, 0.2) \]

**Solution:** The derivative is

\[ y' = \frac{6 - x}{2\sqrt{x}(x + 6)^2}, \]

and its value at the point \( P(4, 0.2) \) is

\[ m_{\text{tan}} = \frac{1}{200}. \]

So an equation of the tangent line to the curve.

\[ y = \frac{\sqrt{x}}{x + 6}, \text{ at the point } P(4, 0.2) \]

is

\[ 200y - x = 36 \]

29. A plane flying horizontally at an altitude of 4 miles (mi) and a speed of 465 miles per hour (mph) passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 10 miles away (horizontal distance) from the station. Round the result to the nearest integer.

**Solution:** We’ll do this in class if someone asks, but it’s a simple right triangle problem. The answer is 432 mph.
30. Find the limit if \( g(x) = x^5 \).

\[
\lim_{x \to 2} \frac{g(x) - g(2)}{x - 2}
\]

**Solution:**

\[
\lim_{x \to 2} \frac{g(x) - g(2)}{x - 2} = \lim_{x \to 2} \frac{x^5 - 2^5}{x - 2}
= \lim_{x \to 2} \frac{x^4 + 2x^3 + 4x^2 + 8x + 16}{x - 2} = 80
\]

31. A company makes computer chips from square wafers of silicon. It wants to keep the side length of a wafer very close to 16 mm. The area is \( A(x) \). Find \( A'(16) \)

**Solution:** \( A(x) = x^2 \) and \( A'(x) = 2x \), so

\[ A'(16) = 32 \text{ mm} \]

32. Calculate \( y' \).

\[ xy^4 + x^2y = x + 3y \]

**Solution:**

\[
\frac{d}{dx} (xy^4 + x^2y) = \frac{d}{dx} (x + 3y)
\]

\[ y^4 + 4xy^3y' + 2xy + x^2 y' = 1 + 3y' \]

\[ y^4 + 2xy - 1 = 3y' - x^2 y' - 4xy^3 y' \]

\[ y^4 + 2xy - 1 = (3 - x^2 - 4xy^3)y' \]

\[
\frac{y^4 + 2xy - 1}{3 - x^2 - 4xy^3} = y'
\]

Your answer might also look like this

\[
y' = \frac{1 - y^4 - 2xy}{4xy^3 + x^2 - 3}
\]
33. Find the first and the second derivatives of the function.

\[ y = \frac{x}{3 - x} \]

**Solution:**

\[
y = \frac{x}{3 - x}
\]

\[
y' = \frac{(3 - x) - x(-1)}{(3 - x)^2} = \frac{3}{(3 - x)^2} = 3(3 - x)^{-2}
\]

\[
y'' = 6(3 - x)^{-3} = \frac{6}{(3 - x)^3}
\]

34. If \( f(t) = \sqrt{4t + 1} \), find \( f''(2) \).

**Solution:**

\[
f(t) = \sqrt{4t + 1} = (4t + 1)^{1/2}
\]

\[
f'(t) = 2(4t + 1)^{-1/2}
\]

\[
f''(t) = -4(4t + 1)^{-3/2}
\]

\[
f''(2) = -\frac{4}{27}
\]

35. If \( y = 2x^3 + 5x \) and \( \frac{dx}{dt} = 3 \), find \( \frac{dy}{dt} \) when \( x = 5 \).

**Solution:**

\[
y = 2x^3 + 5x
\]

\[
\frac{dy}{dt} = (6x^2 + 5) \frac{dx}{dt}
\]

\[
\left. \frac{dy}{dt} \right|_{x=5} = (155) \cdot 3 = 465
\]
36. The volume of a cube is increasing at a rate of 10 cm$^3$ per minute. How fast is the surface area increasing when the length of an edge is 30 cm.

**Solution:** You should know that

$$V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt}.$$  

And since we’re told that the volume of a cube is increasing at a rate of 10 cm$^3$ per minute, we have

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt} \Rightarrow 10 = 3x^2 \frac{dx}{dt} \Rightarrow \frac{10}{3x^2} = \frac{dx}{dt}.$$  

Furthermore, you should know

$$S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x \frac{dx}{dt}.$$  

Using both pieces above, we finally have

$$\left. \frac{dS}{dt} \right|_{x=30} = 12(30) \frac{10}{3(30)^2} = \frac{4}{3} \text{ cm}^2 \text{ per minute}.$$  

37. The point $P(4, 2)$ lies on the curve $y = \sqrt{x}$. If $Q$ is the point $(x, \sqrt{x})$, use your calculator to find the slope of the secant line $PQ$ (correct to six decimal places) for the value $x = 3.99$.

**Solution:** We’re given $P(4, 2)$, and $Q(3.99, \sqrt{3.99})$, the slope is:

$$m_{pq} = \frac{2 - \sqrt{3.99}}{4 - 3.99} \approx 0.250156$$

38. The displacement (in meters) of an object moving in a straight line is given by

$$s = 1 + 2t + \frac{t^2}{4},$$

where $t$ is measured in seconds. Find the average velocity over the time period $[1, 3]$.

**Solution:** Here, $s$ is a function of time $t$.

$$\frac{\Delta s}{\Delta t} = \frac{s(3) - s(1)}{3 - 1} = \frac{6}{2} = 3$$

The average velocity over the time period $[1, 3]$ is 3 meters per second.
39. Find the limit.

\[ \lim_{x \to 5} (9x^2 + 7x + 3) \]

Solution:

\[ \lim_{x \to 5} (9x^2 + 7x + 3) = 263 \]

40. **Optional Problem:** For the limit, illustrate the definition by finding values of \( \delta \) that correspond to \( \varepsilon = 0.25 \).

\[ \lim_{x \to 1} (4 + x - 3x^3) = 2 \]

Solution: Here, using the definition, we have

\[ \lim_{x \to 1} (4 + x - 3x^3) = 2 \]

if for every \( \varepsilon > 0 \), there is a \( \delta > 0 \) such that if \( 0 < |x - 1| < \delta \) then \( |4 + x - 3x^3 - 2| < \varepsilon \). Here, using a graph with \( \varepsilon = 0.25 \), we have to find the \( \delta \) such that if \( |x - 1| < \delta \) then \( |4 + x - 3x^3 - 2| < 0.25 \).

Using a calculator, I found that \( \delta = 0.030212746739 \) so any smaller positive value for \( \delta \) would also work.

![Figure 5: Partial Graph of \( f(x) = |4 + x - 3x^3 - 2| \), \( y = 0.25 \), \( x = 0.97 \), and \( x = 1.03 \).](image-url)
41. Find the limit.

\[ \lim_{x \to 0} \frac{x - 1}{x^2 (x + 5)} \]

**Solution:**

\[ \lim_{x \to 0} \frac{x - 1}{x^2 (x + 5)} = [\infty \text{ or } -\infty] \]

42. If \( 1 \leq f(x) \leq x^2 + 2x + 2 \), for all \( x \), find the limit.

\[ \lim_{x \to -1} f(x) \]

**Solution:** Using the Squeeze Theorem.

\[ \lim_{x \to -1} 1 = \lim_{x \to -1} x^2 + 2x + 2 = 1. \]

Since \( 1 \leq f(x) \leq x^2 + 2x + 2 \), for all \( x \), we have:

\[ \lim_{x \to -1} f(x) = [1] \]

43. Find the limit.

\[ \lim_{x \to 2} \frac{2 - x}{|2 - x|} \]

**Solution:** For \( x > 2 \), that is, for values to the right of 2, we have:

\[ \lim_{x \to 2^+} \frac{2 - x}{|2 - x|} = \lim_{x \to 2^+} \frac{2 - x}{x - 2} = -1, \]

and for \( x < 2 \), that is, for values to the left of 2, we have:

\[ \lim_{x \to 2^-} \frac{2 - x}{|2 - x|} = \lim_{x \to 2^-} \frac{2 - x}{2 - x} = 1. \]

The left and right limits do not agree, therefore the limit

\[ \lim_{x \to 2} \frac{2 - x}{|2 - x|} = \text{Does not exist.} \]
44. Find $y'$.

$$y = \ln \left( x^2 e^x \right)$$

**Solution:**

$$y = \ln \left( x^2 e^x \right) = 2 \ln x + x$$

$$y' = \frac{2}{x} + 1 = \frac{2 + x}{x}$$

45. Find the first and the second derivatives of the function.

$$G(r) = \sqrt{r} + \sqrt[5]{r}$$

**Solution:**

$$G'(r) = \sqrt{r} + \frac{1}{5} r^{-4/5}$$

$$G''(r) = -\frac{1}{4} r^{-3/2} - \frac{4}{25} r^{-9/5}$$

46. Find the equation of the tangent line to the given curve at the specified point.

$$y = 4xe^x, \quad (0, \ 0)$$

**Solution:** First the slope.

$$y' = 4e^x + 4xe^x \quad y'(0) = 4$$

So the equation of the tangent line to $y = 4xe^x$ at $(0, \ 0)$ is

$$y = 4x.$$
47. Find a third-degree polynomial \( Q \) such that \( Q(1) = 2, \ Q'(1) = 7, \ Q''(1) = 14, \) and \( Q'''(1) = 18. \)

**Solution:**

\[
\begin{align*}
Q(x) &= Ax^3 + Bx^2 +Cx + D \\
Q'(x) &= 3Ax^2 + 2Bx + C \\
Q''(x) &= 6Ax + 2B \\
Q'''(x) &= 6A
\end{align*}
\]

Working backwards.

\[
\begin{align*}
Q'''(1) &= 18 = 6A \quad A = 3 \\
Q''(1) &= 14 = 6A + 2B \quad B = -2 \\
Q'(1) &= 7 = 3A + 2B + C \quad C = 2 \\
Q(1) &= 2 = A + B + C + D \quad D = -1
\end{align*}
\]

Bringing it all together.

\[
Q(x) = 3x^3 - 2x^2 + 2x - 1
\]

48. Differentiate the function.

\[
G(u) = \ln \sqrt{\frac{3u + 6}{3u - 6}}
\]

**Solution:**

\[
\begin{align*}
G(u) &= \ln \sqrt{\frac{3u + 6}{3u - 6}} \\
G(u) &= \frac{1}{2} \ln \frac{3u + 6}{3u - 6} \\
G'(u) &= \frac{1}{2} \cdot \frac{3u - 6}{3u + 6} \cdot \frac{3(3u - 6) - 3(3u + 6)}{(3u - 6)^2} \\
&= -\frac{2}{u^2 - 4}
\end{align*}
\]
49. Evaluate.
\[
\lim_{x \to 1} \frac{x^{1000} - 1}{x - 1}
\]

**Solution:**
\[
\lim_{x \to 1} \frac{x^{1000} - 1}{x - 1} = \lim_{x \to 1} (x^{999} + x^{998} + x^{997} + \cdots + x^2 + x + 1) = 1000
\]

50. If a ball is thrown vertically upward with a velocity of 200 ft/s, then its height after \( t \) seconds is \( s = 200t - 10t^2 \). What is the maximum height reached by the ball?

**Solution:** The derivative is
\[
s' = 200 - 20t,
\]
and the ball will reach its maximum height when it stops moving upward, hence the velocity is zero. The derivative is velocity and it is zero when \( t = 10 \) seconds. The height of the ball when \( t = 10 \) seconds, is \( 1000 \) feet.

51. The curve with equation \( y^2 = 17x^4 - x^2 \) is called a *kampyle of Eudoxus*. Find an equation of the tangent line to this curve at the point \((1, 4)\).

**Solution:**
\[
y^2 = 17x^4 - x^2 \Rightarrow y' = \frac{34x^3 - x}{y} \Rightarrow y'(1, 4) = \frac{33}{4}
\]
So the equation is:
\[
y - 4 = \frac{33}{4} (x - 1)
\]
52. Use logarithmic differentiation to find the derivative of the function.

\[ f(x) = \sqrt[3]{\frac{x^2 + 1}{x^2 - 1}} \]

**Solution:**

\[
\frac{d}{dx} [\ln (f(x))] = \frac{d}{dx} \left[ \frac{1}{3} \ln (x^2 + 1) - \frac{1}{3} \ln (x^2 - 1) \right]
\]

\[
\frac{f'(x)}{f(x)} = \frac{2x}{3(x^2 + 1)} - \frac{2x}{3(x^2 - 1)}
\]

\[
\frac{f'(x)}{f(x)} = -\frac{4x}{3(x^4 - 1)}
\]

\[
f'(x) = -\frac{4x}{3(x^4 - 1)} \cdot \sqrt[3]{\frac{x^2 + 1}{x^2 - 1}}
\]

53. Find the limit.

\[
\lim_{\theta \to 0} \frac{\sin (\sin \theta)}{\sec \theta}
\]

**Solution:**

\[
\lim_{\theta \to 0} \frac{\sin (\sin \theta)}{\sec \theta} = 0
\]