2005 A. Shloming Mathematics Prize Examination
Essex County College—Division of Mathematics and Physics
Printed[1] April 1, 2005

Name: _____

Signature: _____

Select one answer for each of the following twenty problems. **Five points** for each correct answer. **Negative one point** for each incorrect answer. **One point** for each problem left blank. No calculators or cellular phones allowed.

1. If the points $(1, y_1)$ and $(-1, y_2)$ lie on the graph of $y = ax^2 + bx + c$, and $y_1 - y_2 = -6$, then b equals

Answer **a**

(a) -3

(b) 0

(c) 3

(d) $\sqrt{ac}$

(e) $\dfrac{a+c}{2}$

2. The equation $2^{2x} - 8 \cdot 2^x + 12 = 0$ is satisfied by:

Answer **d**

(a) $\log 3$

(b) $\dfrac{1}{2}\log 6$

(c) $1 + \log\left(\dfrac{3}{2}\right)$

(d) $1 + \dfrac{\log 3}{\log 2}$

(e) $\dfrac{\log 2}{\log 3}$

[1]This document was prepared by Ron Bannon using LaTeX 2_ε and was based on set of problems kindly submitted by Robert Shloming, PhD. Source and pdf are available by emailing a request to ron.bannon@gmail.com.

3. The fourth power of $\sqrt{1+\sqrt{1+\sqrt{1}}}$ is

Answer **e**

(a) $\sqrt{2}+\sqrt{3}$

(b) $\dfrac{7+3\sqrt{5}}{2}$

(c) $1+2\sqrt{3}$

(d) 3

(e) $3+2\sqrt{2}$

4. A square and a circle have equal perimeters. The ratio of the area of the circle to the area of the square is

Answer **a**

(a) $\dfrac{4}{\pi}$

(b) $\dfrac{\pi}{\sqrt{2}}$

(c) $\dfrac{4}{1}$

(d) $\dfrac{\sqrt{2}}{\pi}$

(e) $\dfrac{\pi}{4}$

5. If $f(x)=\dfrac{x^4+x^2}{x+1}$, then $f(i)$, where $i=\sqrt{-1}$, is equal to

Answer **d**

(a) $i+1$

(b) 1

(c) -1

(d) 0

(e) $-1-i$

6. If $a = \log_8 225$ and $b = \log_2 15$, then

Answer **b**

(a) $a = \dfrac{b}{2}$

(b) $a = \dfrac{2b}{3}$

(c) $a = b$

(d) $b = \dfrac{a}{2}$

(e) $a = \dfrac{3b}{2}$

7. If the point $(x, -4)$ lies on the straight line joining the points $(0, 8)$ and $(-4, 0)$, then x is equal to

Answer **e**

(a) -2

(b) 2

(c) -8

(d) 6

(e) -6

8. If the line $y = mx + 1$ intersects the ellipse $x^2 + 4y^2 = 1$ exactly once, then m^2 is

Answer **c**

(a) $\dfrac{1}{2}$

(b) $\dfrac{2}{3}$

(c) $\dfrac{3}{4}$

(d) $\dfrac{4}{5}$

(e) $\dfrac{5}{6}$

9. The sum of the squares of the roots of $x^2 + 2hx = 3$ is 10. The absolute value of h is

Answer **a**

(a) 1

(b) $\frac{1}{2}$

(c) $\frac{3}{2}$

(d) 2

(e) none of these

10. The area of a sector of a circle with radius 5 cm is 10 cm^2. The central angle, measured in radians, is

Answer **d**

(a) $\frac{\pi}{2}$

(b) 1

(c) $\frac{\pi}{3}$

(d) $\frac{4}{5}$

(e) none of these

11. If $0 < \alpha < \frac{\pi}{2}$, and $\sin\alpha + 1 = 2\sqrt{1 - \sin^2\alpha}$, then $\sin\alpha$ is

Answer **d**

(a) $\frac{4}{5}$

(b) 1

(c) -1

(d) $\frac{3}{5}$

(e) 0

12. If one root of $x^3 - 5x^2 + 5x - 1 = 0$ is $2 - \sqrt{3}$, then the sum of the other two roots is

Answer **c**

(a) $-7 + \sqrt{3}$

(b) $-1 + \sqrt{3}$

(c) $3 + \sqrt{3}$

(d) $-3 + \sqrt{3}$

(e) 5

13. If $2^{x+1} + 2^x = 3^{y+2} - 3^y$, where x and y are integers, then the value of x is

Answer **b**

(a) 0

(b) 3

(c) -1

(d) 1

(e) 2

14. The value of $\lim_{\theta \to 0} \dfrac{\sin^2 \theta \cot \theta}{\theta}$

Answer **b**

(a) π

(b) 1

(c) 0

(d) undefined

(e) -1

15. What is the slope of the tangent line to $y = x \sin x$ at $x = \pi$. Answer **a**

(a) $-\pi$

(b) π

(c) 0

(d) 1

(e) -1

16. $\cot(10) + \tan(5)$ is Answer **b**

(a) $\csc(5)$

(b) $\csc(10)$

(c) $\sec(5)$

(d) $\sec(10)$

(e) $\sin(15)$

17. Find the sum of the roots of $\tan^2 x - 9\tan x + 1 = 0$ that are between $x = 0$ and $x = 2\pi$ radians. Answer **d**

(a) $\frac{\pi}{2}$

(b) π

(c) $\frac{3\pi}{2}$

(d) 3π

(e) 4π

18. The consecutive angles of a trapezoid form an arithmetic sequence. If the smallest angle is $75°$, then the largest angle is

Answer **c**

(a) $95°$

(b) $100°$

(c) $105°$

(d) $110°$

(e) $115°$

19. If $f(x) = ax^2 - \sqrt{2}$, for all $a > 0$ and $f\left(f\left(\sqrt{2}\right)\right) = -\sqrt{2}$, then a is

Answer **d**

(a) $\dfrac{2-\sqrt{2}}{2}$

(b) $\dfrac{1}{2}$

(c) $2-\sqrt{2}$

(d) $\dfrac{\sqrt{2}}{2}$

(e) $\dfrac{2+\sqrt{2}}{2}$

20. If $x, y > 0$, $\log_y x + \log_x y = \dfrac{10}{3}$ and $xy = 144$, then $\dfrac{x+y}{2}$ is

Answer **b**

(a) $12\sqrt{2}$

(b) $13\sqrt{3}$

(c) 24

(d) 30

(e) 36