

2006 A. Shloming Mathematics Prize Examination
Essex County College—Division of Mathematics and Physics
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Name: _____

Signature: _____

If the question has choices, select one answer; if the question is open ended, write your final answer on the line provided. **Five points** for each correct answer. No calculators are allowed, and the use of cellular phones is strictly forbidden.

1. What is the remainder when $x^{51} + 51$ is divided by $x + 1$?

Answer **d**

(a) 0

(b) 1

(c) 49

(d) 50

(e) 51

2. If the operation $x \otimes y$ is defined by $x \otimes y = (x + 1)(y + 1) - 1$, then which one of the following is false?

Answer **b**

(a) $x \otimes y = y \otimes x$ for all real x and y .

(b) $x \otimes (y + z) = (x \otimes y) + (x \otimes z)$ for all real x , y , and z .

(c) $(x - 1) \otimes (x + 1) = (x \otimes x) - 1$ for all real x .

(d) $x \otimes 0 = x$ for all real x .

(e) $x \otimes (y \otimes z) = (x \otimes y) \otimes z$ for all real x , y and z .

¹This document was prepared by Ron Bannon using L^AT_EX 2_ε and was based on a variety of sources.

3. Solve for x .

$$(\log_3 x)(\log_x 5) = \log_3 5$$

Answer $x > 0, x \neq 1$

Answer: _____

4. The coefficient of x^7 in the polynomial expansion of $(1 + 2x - x^2)^4$ is

Answer **a**

(a) -8

(b) 12

(c) 6

(d) -12

(e) none of these

5. The perimeter of a semicircular region, measured in centimeters, is numerically equal to its area, measured in square centimeters. The radius of the semicircle, measured in centimeters, is

Answer **e**

(a) π

(b) $\frac{2}{\pi}$

(c) 1

(d) $\frac{1}{2}$

(e) $\frac{4}{\pi} + 2$

6. If $p \geq 5$ is a prime number, then 24 divides $p^2 - 1$ without remainder

Answer **c**

- (a) never
- (b) sometimes
- (c) always
- (d) only if $p = 5$
- (e) none of these

7. If $a > 1$, $b > 1$ and $p = \frac{\log_b(\log_b a)}{\log_b a}$, then a^p equals

Answer **d**

- (a) 1
- (b) b
- (c) $\log_a b$
- (d) $\log_b a$
- (e) $a^{\log_b a}$

8. Find the smallest integral value of k so that $2x(kx - 4) - x^2 + 6 = 0$ has no real roots.

Answer **2**

Answer: _____

9. A sector with an acute central angle θ is cut from a circle with radius 6. The radius of the circle circumscribed about the sector is

Answer **d**

(a) $3 \cos \theta$

(b) $3 \sec \theta$

(c) $3 \cos \frac{\theta}{2}$

(d) $3 \sec \frac{\theta}{2}$

(e) 3

10. Given $g(x) = 1 - x^2$, and $f(g(x)) = \frac{1 - x^2}{x^2}$, $x \neq 0$. Find $f\left(\frac{1}{2}\right)$.

Answer **1**

Answer: _____

11. Two different prime numbers between four and eighteen are chosen. When their sum is subtracted from their product, which of the following numbers could be obtained?

Answer **c**

(a) 21

(b) 60

(c) 119

(d) 180

(e) 231

12. If $0 < \theta < \frac{\pi}{2}$ and $10 \sin \theta = z$, what is $\tan \theta$ in terms of z ?

Answer **a**

(a) $\frac{z}{\sqrt{100 - z^2}}$

(b) $\frac{10}{\sqrt{z^2 - 100}}$

(c) $\frac{\sqrt{100 - z^2}}{10}$

(d) $\frac{\sqrt{z^2 - 100}}{10}$

(e) $\frac{\sqrt{100 - z^2}}{z}$

13. Find the smallest prime number that divides the sum $3^{11} + 5^{13}$.

Answer **2**

Answer: _____

14. What is the x -intercept of the graph of $y = \frac{1}{8}x^{3/2} - 8$?

Answer **d**

(a) -16

(b) -8

(c) 16^{-1}

(d) 16

(e) 512

15. If θ is an acute angle and $\sin \frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$, then $\tan \theta$ is equal to

Answer e

(a) x

(b) $\frac{1}{x}$

(c) $\frac{\sqrt{x-1}}{x+1}$

(d) $\frac{\sqrt{x^2-1}}{x}$

(e) $\sqrt{x^2-1}$

16. In the xy -plane, the graph of $y = x^2 + bx + c$ is symmetric about the line $x = 3$ and passes through the point $(5, 2)$. What is the value of c ?

Answer 7

Answer: _____

17. The sum of the integers between 50 and 350 which end in 1, is

Answer a

(a) 5880

(b) 5539

(c) 5280

(d) 4877

(e) 4566

18. If a and b are numbers such that $\ln a = 2.1$ and $\ln b = 1.4$, what is the value of $\ln \left(\frac{a^2}{b} \right)$?

Answer 2.8

Answer: _____

19. A chord which is a perpendicular bisector of a circle with radius 12 inches, has length

Answer d

(a) $3\sqrt{3}$

(b) 27

(c) $6\sqrt{3}$

(d) $12\sqrt{3}$

(e) none of these

20. What is the range of $f(x)$?

$$f(x) = \begin{cases} 3 \sin x & \text{for } x < 0 \\ \sqrt{x} & \text{for } x \geq 0 \end{cases}$$

Answer a

(a) $[-3, \infty)$

(b) $[0, \infty)$

(c) $[-3, 0)$

(d) $[-3, 3)$

(e) $(-\infty, \infty)$

21. The domain of the function f is $\{x : -1 \leq x \leq 5\}$. If $g(x) = 2f(-x)$, what is the domain of the function g ?

Answer **b**

(a) $\{x : -10 \leq x \leq 2\}$

(b) $\{x : -5 \leq x \leq 1\}$

(c) $\{x : -2 \leq x \leq 10\}$

(d) $\{x : -1 \leq x \leq 5\}$

(e) $\{x : 1 \leq x \leq 5\}$

22. The fraction

$$\frac{2(\sqrt{2} + \sqrt{6})}{3\sqrt{2 + \sqrt{3}}}$$

Answer **d**

(a) $\frac{2\sqrt{2}}{3}$

(b) 1

(c) $\frac{2\sqrt{3}}{3}$

(d) $\frac{4}{3}$

(e) $\frac{16}{9}$

23. If x , y , and z are positive numbers satisfying

$$x + \frac{1}{y} = 4, \quad y + \frac{1}{z} = 1, \quad \text{and} \quad z + \frac{1}{x} = \frac{7}{3},$$

then xyz is

Answer **b**

(a) $\frac{2}{3}$

(b) 1

(c) $\frac{4}{3}$

(d) 2

(e) $\frac{7}{3}$