Name: _____

Signature: _____

Show all work *clearly* and in *order*, and $\boxed{\text{box your final answers.}}$ Justify your answers algebraically whenever possible. You have at most 80 minutes to take this 100 point exam. No cellular phones or calculators allowed.

- **Part 1: Short multi-step problems. Two points may be given for one error, no credit for more than one error. Four points each. All answers must be in simplest form.**

    1. Find the y-intercept of the line whose equation is $3x - 5y = 15$.

        Solution: Set $x = 0$ and solve for y.

        $$x = 0 \quad \Rightarrow \quad -5y = 15 \quad \Rightarrow \quad y = -3 \quad \Rightarrow \quad \boxed{(0,\ -3)}$$

    2. Simplify: $\dfrac{27x^5 (2y - 3)^3}{36x^2 (2y - 3)^2}$.

        Solution:
        $$\frac{27x^5 (2y - 3)^3}{36x^2 (2y - 3)^2} = \boxed{\frac{3x^3 (2y - 3)}{4}}$$

    3. Factor: $x^2 - 5x + 6$.

        Solution:
        $$x^2 - 5x + 6 = \boxed{(x - 2)(x - 3)}$$

    4. Find the slope of the line whose equation is $3x + 4y = 7$.

        Solution: Solve for y, and the slope will be equal to the numerical coefficient of x.

        $$\begin{aligned} 3x + 4y &= 7 \\ 4y &= -3x + 7 \\ y &= -\frac{3}{4}x + \frac{7}{4} \\ m &= \boxed{-\frac{3}{4}} \end{aligned}$$

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[1]This document was prepared by Ron Bannon using LaTeX 2_ε.

5. Simplify: $\dfrac{3x-2}{2x-5} - \dfrac{5x-7}{2x-5}$.

   Solution:

   $$\frac{3x-2}{2x-5} - \frac{5x-7}{2x-5} = \frac{(3x-2)-(5x-7)}{2x-5} = \frac{3x-2-5x+7}{2x-5} = \boxed{\frac{5-2x}{2x-5}} = \boxed{-1}$$

6. List one ordered pair solution of the equation $5y = 6x - 11$.

   Solution: There are an infinite number of correct answers.

   $$\boxed{(1,\ -1)}$$

7. Simplify: $\dfrac{27x^4y^3}{24a^4b^2} \div \dfrac{45x^3y^4}{16a^7b}$.

   Solution:

   $$\frac{27x^4y^3}{24a^4b^2} \div \frac{45x^3y^4}{16a^7b} = \frac{27x^4y^3}{24a^4b^2} \cdot \frac{16a^7b}{45x^3y^4} = \boxed{\frac{2a^3x}{5by}}$$

8. Find the x-intercept of the line whose equation is $7x - 2y = 21$.

   Solution: Set $y = 0$ and solve for x.

   $$y = 0 \quad \Rightarrow \quad 7x = 21 \quad \Rightarrow \quad x = 3 \quad \Rightarrow \quad \boxed{(3,\ 0)}$$

9. Simplify: $\dfrac{9x^3}{16y^2} \cdot \dfrac{12y^3}{27x^4}$.

   Solution:

   $$\frac{9x^3}{16y^2} \cdot \frac{12y^3}{27x^4} = \boxed{\frac{y}{4x}}$$

10. Is $(-4, -3)$ a solution of $2(y - 5) = 3x$?

    Solution:

    $$2(-3 - 5) = 3(-4) \quad \Rightarrow \quad 2(-8) = -12 \quad \Rightarrow \quad -16 = -12 \quad \Rightarrow \quad \boxed{\text{No.}}$$

- **Part 2: Short multi-step problems. Three points may be given for one error, no credit for more than one error. Six points each. All answers must be in simplest form.**

1. Factor: $15x^2 + x - 6$.

   Solution:
   $$15x^2 + x - 6 = \boxed{(5x - 3)(3x + 2)}$$

2. The sum of two numbers is 11. The sum of the squares of these two numbers is 73. Find the two numbers.

   Solution: Let the two numbers be x and $11 - x$.
   $$\begin{aligned}
   x^2 + (11 - x)^2 &= 73 \\
   x^2 + 121 - 22x + x^2 &= 73 \\
   2x^2 - 22x + 48 &= 0 \\
   x^2 - 11x + 24 &= 0 \\
   (x - 8)(x - 3) &= 0
   \end{aligned}$$

   $\boxed{\text{The two numbers are 8 and 3.}}$

3. Simplify: $\dfrac{x^2 + x - 6}{x^2 + 3x - 10} \div \dfrac{x^2 - x - 6}{x^2 + 7x + 10}$.

   Solution:
   $$\begin{aligned}
   \frac{x^2 + x - 6}{x^2 + 3x - 10} \div \frac{x^2 - x - 6}{x^2 + 7x + 10} &= \frac{x^2 + x - 6}{x^2 + 3x - 10} \cdot \frac{x^2 + 7x + 10}{x^2 - x - 6} \\
   &= \frac{(x + 3)(x - 2)}{(x + 5)(x - 2)} \cdot \frac{(x + 5)(x + 2)}{(x - 3)(x + 2)} \\
   &= \boxed{\frac{x + 3}{x - 3}}
   \end{aligned}$$

4. Simplify: $\dfrac{x^2 - 8x + 7}{x^2 + 3x - 4} \cdot \dfrac{x^2 + 3x - 10}{x^2 - 9x + 14}$.

   Solution:
   $$\frac{x^2 - 8x + 7}{x^2 + 3x - 4} \cdot \frac{x^2 + 3x - 10}{x^2 - 9x + 14} = \frac{(x - 7)(x - 1)}{(x - 1)(x + 4)} \cdot \frac{(x - 2)(x + 5)}{(x - 7)(x - 2)} = \boxed{\frac{x + 5}{x + 4}}$$

5. Simplify: $\dfrac{2x-1}{x^2-25} - \dfrac{2}{x-5}$.

   Solution:

$$
\begin{aligned}
\frac{2x-1}{x^2-25} - \frac{2}{x-5} &= \frac{2x-1}{(x+5)(x-5)} - \frac{2}{x-5} \\
&= \frac{2x-1}{(x+5)(x-5)} - \frac{2}{x-5} \cdot \frac{x+5}{x+5} \\
&= \frac{(2x-1) - 2(x+5)}{(x+5)(x-5)} \\
&= \frac{2x-1-2x-10}{(x+5)(x-5)} \\
&= \boxed{\frac{-11}{(x+5)(x-5)}} = \boxed{\frac{-11}{x^2-25}}
\end{aligned}
$$

6. Solve for x.

$$(x+5)(x-1) = 16$$

   Solution:

$$
\begin{aligned}
(x+5)(x-1) &= 16 \\
x^2 + 4x - 5 &= 16 \\
x^2 + 4x - 21 &= 0 \\
(x+7)(x-3) &= 0
\end{aligned}
$$

   This equation has $\boxed{x = -7 \text{ or } x = 3}$ as a solution.

7. The length of a rectangular garden is four feet longer than three its width. If the area of this rectangular garden is 95 square feet, determine the length and width of this rectangular garden.

   Solution: Area of a rectangle is $w \cdot l$. Let $l = 3w + 4$.

$$
\begin{aligned}
95 &= w \cdot (3w + 4) \\
95 &= 3w^2 + 4w \\
0 &= 3w^2 + 4w - 95 \\
0 &= (3w + 19)(w - 5)
\end{aligned}
$$

   So the $\boxed{\text{width of the rectangle is 5 feet and the length is 19 feet}}$.

8. Factor by grouping: $xy + 6x - 4y - 24$

   Solution:

$$xy + 6x - 4y - 24 = x(y+6) - 4(y+6) = \boxed{(y+6)(x-4)}$$

4

9. Solve for x.

$$\frac{2x}{x-5} = 4 + \frac{3x}{x-5}$$

Solution: The LCD is $(x-5)$.

$$
\begin{aligned}
\frac{2x}{x-5} \cdot (x-5) &= 4 \cdot (x-5) + \frac{3x}{x-5} \cdot (x-5) \\
2x &= 4x - 20 + 3x \\
2x &= 7x - 20 \\
20 &= 5x \\
4 &= x
\end{aligned}
$$

This equation has $\boxed{x = 4}$ as a solution.

10. Graph $5x - 3y = 2$.

Solution: There are an infinite number of points on this line, and you'll need to find at least two of these points to graph the line. For example, both $(1,\ 1)$ and $(-2,\ -4)$ are easy to find and plot.
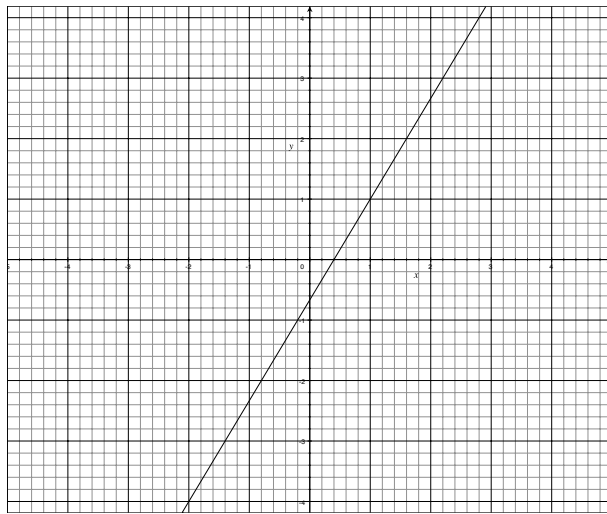

Figure 1: Graph of $5x - 3y = 2$.