MTH 121 — Fall — 2004

Essex County College — Division of Mathematics

MIT Extra Credit Questions 2 and 4 — Version α [1] — Created November 2, 2004

Name: ______________________________

Signature: ______________________________

- **Problem 2 from MIT exam** (5 points): The point $P = (0, 1)$ lies on two distinct lines tangent to the parabola with equation $y = x^2 + 2$. Find the equations of *both* tangent lines. Show all work and box the final answer.

  Solution:

$$\begin{aligned} y &= x^2 + 2 \\ y' &= 2x \\ 2x &= \frac{y-1}{x-0} \\ 2x &= \frac{x^2+2-1}{x-0} \\ 2x\,(x) &= x^2 + 1 \\ 2x^2 &= x^2 + 1 \\ x^2 - 1 &= 0 \\ (x-1)(x+1) &= 0 \\ x = \pm 1 & \end{aligned}$$

  For the equation of the first tangent line (at the point on the parabola where $x = 1$) we have a point on the parabola $(1, 3)$ with slope $m = 2$.

$$\boxed{y - 3 = 2\,(x - 1)}$$

  or

$$\boxed{y = 2x + 1}$$

  For the equation of the second tangent line (at the point on the parabola where $x = -1$) we have a point on the parabola $(-1, 3)$ with slope $m = -2$.

$$\boxed{y - 3 = -2\,(x + 1)}$$

  or

$$\boxed{y = -2x + 1}$$

[1] This document was prepared by Ron Bannon using LaTeX.

- **Problem 2 from MIT exam** (5 points): The point $T = \left(-\frac{4\sqrt{10}}{5}, \sqrt{10}\right)$ lies on the ellipse with equation

$$9(x+y)^2 + (x-y)^2 = 36.$$

Using implicit differentiation, determine the equation of the tangent line to the ellipse at the point T; *do not simply find the slope of the tangent line*, you must write the equation of the tangent line. Show all work and box the final answer.

Solution:

$$\begin{aligned}
9(x+y)^2 + (x-y)^2 &= 36 \\
18(x+y)\left(1+y'\right) + 2(x-y)\left(1-y'\right) &= 0 \\
18(x+y) + 18(x+y)y' + 2(x-y) - 2(x-y)y' &= 0 \\
18(x+y) + 2(x-y) &= 2(x-y)y' - 18(x+y)y' \\
20x + 16y &= [2(x-y) - 18(x+y)]\,y' \\
20x + 16y &= -(16x+20y)\,y' \\
\frac{20x+16y}{-(16x+20y)} &= y' \\
-\frac{5x+4y}{4x+5y} &= y'
\end{aligned}$$

Use y' at the point T to find the slope.

$$y' = -\frac{5\left(-\frac{4\sqrt{10}}{5}\right) + 4\sqrt{10}}{4\left(-\frac{4\sqrt{10}}{5}\right) + 5\sqrt{10}} = 0$$

For the equation of the tangent line we have a point on the ellipse $\left(-\frac{4\sqrt{10}}{5}, \sqrt{10}\right)$ with slope $m = 0$.

$$\boxed{y - \sqrt{10} = 0\left(x + \frac{4\sqrt{10}}{5}\right)}$$

or

$$\boxed{y = \sqrt{10}}$$