MTH 121 — Fall — 2004

Essex County College — Division of Mathematics

MIT Extra Credit Errata — Version α [1] — Created November 2, 2004

Name: ______________________________

Signature: ______________________________

- **Extra Credit** (5 points) Let $Q = (x_0, y_0)$ and $R = (x_1, y_1)$ be two distinct points on the parabola whose equation is $y = x^2 + 2$. There is a unique point $S = (x_2, y_2)$ which lies on both the tangent line at Q and on the tangent line at R. Show that for *every* pair of distinct points Q and R on the parabola, $2x_2 = x_0 + x_1$, *i.e.* the line passing through S parallel to the y-axis bisects the line segment QR.

  Solution: We know that $f'(x) = y' = 2x$, and that the slope of the tangent line at Q and R, respectively, is:

$$2x_0 = \frac{y_2 - y_0}{x_2 - x_0} \tag{1}$$

$$2x_1 = \frac{y_2 - y_1}{x_2 - x_1} \tag{2}$$

  Now, we need to show that this implies $2x_2 = x_0 + x_1$, first by solving these two equations for y_2, and the fact that $y_0 = x_0^2 + 2$ and $y_1 = x_1^2 + 2$.

$$\begin{aligned} 2x_0(x_2 - x_0) &= y_2 - y_0 \\ 2x_0(x_2 - x_0) &= y_2 - x_0^2 - 2 \\ 2x_0(x_2 - x_0) + x_0^2 + 2 &= y_2 \end{aligned}$$

  and

$$\begin{aligned} 2x_1(x_2 - x_1) &= y_2 - y_1 \\ 2x_1(x_2 - x_1) &= y_2 - x_1^2 - 2 \\ 2x_1(x_2 - x_1) + x_1^2 + 2 &= y_2 \end{aligned}$$

  Since $y_2 = y_2$, we get:

$$\begin{aligned} 2x_0(x_2 - x_0) + x_0^2 + 2 &= 2x_1(x_2 - x_1) + x_1^2 + 2 \\ 2x_0(x_2 - x_0) + x_0^2 &= 2x_1(x_2 - x_1) + x_1^2 \\ 2x_0(x_2 - x_0) - 2x_1(x_2 - x_1) &= x_1^2 - x_0^2 \\ 2x_0x_2 - 2x_0^2 - 2x_1x_2 + 2x_1^2 &= x_1^2 - x_0^2 \\ 2x_0x_2 - 2x_1x_2 &= x_0^2 - x_1^2 \\ 2x_2(x_0 - x_1) &= (x_0 - x_1)(x_0 + x_1) \\ 2x_2 &= x_0 + x_1 \end{aligned}$$

  Q.E.D.

[1] This document was prepared by Ron Bannon using LaTeX.