MSTM 6033 — Fall — 2004

Essex County College

Exam Extra Credit # 1[1] — October 18, 2004

Name: _____

Signature: _____

Extra credit is worth ten points and will be added to your exam number one score. There is no partial credit. This is due Friday, October 22, 2004 before class starts.[2]

1. Given that $f(x)$ is a cubic polynomial of the form

$$Ax^3 + Bx^2 + Cx + D,$$

and we are given:

$$\begin{aligned} f(1) &= 3, \\ f(-1) &= -9, \\ f'(1) &= 12, \\ f'(-1) &= 20. \end{aligned}$$

Find the values for A, B, C, and D.

Solution: We're given $f(x) = Ax^3 + Bx^2 + Cx + D$, now taking the derivative of $f(x)$ produces $f'(x) = 3Ax^2 + 2Bx + C$. Using the values given:

$$\begin{array}{rcrcrcrcr} f(1) & = & A & + & B & + & C & + & D & = & 3, \\ f(-1) & = & -A & + & B & - & C & + & D & = & -9, \\ f'(1) & = & 3A & + & 2B & + & C & + & 0D & = & 12, \\ f'(-1) & = & 3A & - & 2B & + & C & + & 0D & = & 20. \end{array}$$

Now, you'll need to recall how to solve a system of equations (MTH-119). Using matrices is a standard method, and can easily be done on a calculator using the `rref` (reduced row echelon from) command:

$$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & 3 \\ -1 & 1 & -1 & 1 & -9 \\ 3 & 2 & 1 & 0 & 12 \\ 3 & -2 & 1 & 0 & 20 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

Which, of course gives: $A = 5$, $B = -2$, $C = 1$, and $D = -1$. You should of course check, that if $f(x) = 5x^3 - 2x^2 + x - 1$, then $f(1) = 3$, $f(-1) = -9$, $f'(1) = 12$, and $f'(-1) = 20$. I hope it does!

---

[1] This document was prepared by Ron Bannon using LaTeX. Source and pdf are available by emailing a request to `rbannon@mac.com`.

[2] Late work will not be accepted.