MTH 121 — Fall — 2004

Essex County College — Division of Mathematics

Quiz # 7[1] — November 11, 2004

Name: ________________________________

Signature: ________________________________

Show all work clearly and in order, and box your final answers. Justify your answers algebraically whenever possible. You have 20 minutes to take this 10 point quiz. When you do use your calculator, sketch all relevant graphs and write down all relevant mathematics. **Do only one of the following two problems.**

1. The volume[2] of a cube is increasing at a rate of 10 cm^3/min. How fast is the surface area[3] increasing when the length of an edge is 30 cm?

   Solution: We're given a rate:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{10\ \text{cm}^3}{\text{min}}$$

   We're asked for a rate when the edge is 30 cm, which means the surface area is $6\,(30)^2 = 5400\ \text{cm}^2$:

$$\left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{S=5400} = ?$$

   We can solve the surface area function for x, keeping in mind that $x \geq 0$:

$$S = 6x^2 \quad \rightarrow \quad x = \sqrt{\frac{S}{6}}$$

   Now write V in terms of S:

$$V = x^3 = \left(\sqrt{\frac{S}{6}}\right)^3 = \left(\frac{S}{6}\right)^{\frac{3}{2}}$$

   Differentiate $V = \left(\frac{S}{6}\right)^{\frac{3}{2}}$ with respect to t.

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{3}{2}\left(\frac{S}{6}\right)^{\frac{1}{2}} \frac{1}{6}\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{1}{4}\sqrt{\frac{S}{6}}\frac{\mathrm{d}S}{\mathrm{d}t}$$

   Substitute, $S = 5400$, $\frac{\mathrm{d}V}{\mathrm{d}t} = 10$ and solve for $\frac{\mathrm{d}S}{\mathrm{d}t}$:

$$10 = \frac{1}{4}\sqrt{\frac{5400}{6}}\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{15}{2}\frac{\mathrm{d}S}{\mathrm{d}t} \quad \rightarrow \quad \frac{\mathrm{d}S}{\mathrm{d}t} = \boxed{\frac{4}{3}\,\frac{\text{cm}^2}{\text{min}}}$$

   This is not the only way to solve this problem, but the answer will always be the same.

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[1] This document was prepared by Ron Bannon using LaTeX.

[2] $V = x^3$

[3] $S = 6x^2$

2. Find the linearization of

$$f(x) = \sqrt{25 - x^2}$$

near $x = 3$. Use this linear approximating function to approximate the $f(3.01)$.

Find the derivative of $f(x)$:

$$f'(x) = \frac{1}{2}\left(25 - x^2\right)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{25 - x^2}}$$

Evaluate the derivative of $x = 3$:

$$f'(3) = \frac{-3}{\sqrt{25 - 3^2}} = -\frac{3}{4}$$

The linearization of $f(x)$ at $x = 3$ and $y = f(3) = 4$:

$$y - 4 = -\frac{3}{4}(x - 3) \quad \rightarrow \quad y = \frac{25 - 3x}{4}$$

Using this to approximate $f(3.01)$;

$$f(3.01) \approx \frac{25 - 3(3.01)}{4} = \boxed{3.9925}$$