MTH 121 — Fall — 2004

Essex County College — Division of Mathematics

Quiz # 10[1] — December 3, 2004

Name: ______________________________

Signature: ______________________________

Show all work clearly and in order, and box your final answers. Justify your answers algebraically whenever possible. You have 20 minutes to take this 10 point quiz. When you do use your calculator, sketch all relevant graphs and write down all relevant mathematics.

1. Evaluate

$$\int_{-1}^{2} \left(x^2+1\right) \mathrm{dx}$$

   (a) By using the Fundamental Theorem of Calculus.
   Solution:

$$\int_{-1}^{2} \left(x^2+1\right) \mathrm{dx} = \left(\frac{\mathrm{x}^3}{3}+\mathrm{x}\right)\Bigg|_{\mathrm{x}=-1}^{\mathrm{x}=2} = \left(\frac{8}{3}+2\right) - \left(-\frac{1}{3}-1\right) = \boxed{6}$$

   (b) By setting up an expression for $\int_{-1}^{2} \left(x^2+1\right) \mathrm{dx}$ as a limit of sums.
   Solution: First. find and simplify the n^{th} partial sum.

$$\begin{aligned}
S_n &= \frac{3}{n}\cdot f\left(-1+1\cdot\frac{3}{n}\right) + \frac{3}{n}\cdot f\left(-1+2\cdot\frac{3}{n}\right) + \cdots + \frac{3}{n}\cdot f\left(-1+n\cdot\frac{3}{n}\right) \\
S_n &= \sum_{i=1}^{n}\left(\frac{3}{n}\cdot f\left(-1+\frac{3i}{n}\right)\right) \\
S_n &= \sum_{i=1}^{n}\left(\frac{3}{n}\left[\left(-1+\frac{3i}{n}\right)^2+1\right]\right) \\
S_n &= \sum_{i=1}^{n}\left(\frac{3}{n}\left[2-\frac{6i}{n}+\frac{9i^2}{n^2}\right]\right) \\
S_n &= \sum_{i=1}^{n}\left(\frac{6}{n}-\frac{18i}{n^2}+\frac{27i^2}{n^3}\right) \\
S_n &= \frac{6}{n}\cdot n - \frac{18}{n^2}\cdot\frac{n\,(n+1)}{2} + \frac{27}{n^3}\cdot\frac{n\,(n+1)\,(2n+1)}{6} \\
S_n &= 6-9\left(1+\frac{1}{n}\right)+\frac{9}{2}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)
\end{aligned}$$

   Now, take the limit as n goes to infinity.

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[6-9\left(1+\frac{1}{n}\right)+\frac{9}{2}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)\right] = \boxed{6}$$

[1] This document was prepared by Ron Bannon using LaTeX.