MTH 121 — Fall — 2004

Essex County College

Extra Credit # 8[1] — November 23, 2004

Name: ________________________________

Signature: ________________________________

$\boxed{\text{Box your final answer}}$ and show all relevant work.

1. Find all points on the function where a global maximum occurs.

$$f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x-2|}$$

Solution:

$$f(x) = \begin{cases} \dfrac{1}{1-x} + \dfrac{1}{3-x}, & \text{if } x < 0; \\ \dfrac{1}{1+x} + \dfrac{1}{3-x}, & \text{if } 0 \le x < 2; \\ \dfrac{1}{1+x} + \dfrac{1}{x-1}, & \text{if } x \ge 2. \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{1}{(1-x)^2} + \dfrac{1}{(3-x)^2}, & \text{if } x < 0; \\ \dfrac{-1}{(1+x)^2} + \dfrac{1}{(3-x)^2}, & \text{if } 0 < x < 2; \\ \dfrac{-1}{(1+x)^2} - \dfrac{1}{(x-1)^2}, & \text{if } x > 2. \end{cases}$$

$f'(x) > 0$ for $x < 0$, $f'(x) < 0$ for $x > 2$, and $f'(x) = \dfrac{8(x-1)}{(1+x)^2(3-x)^2}$ for $0 < x < 2$.

Visually we need to analyze the first derivative on the number line:

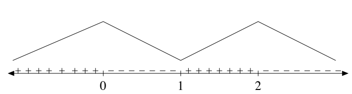

Figure 1: Analysis of $f'(x)$

[1]This document was prepared by Ron Bannon using LaTeX. Source and pdf are available by emailing a request to `rbannon@mac.com`.

It should be clear that two peaks are formed at $x = 0$ and $x = 2$. Evaluating $f(0) = \frac{4}{3}$ and $f(2) = \frac{4}{3}$, gives two points:

$$\boxed{\left(0, \frac{4}{3}\right), \quad \left(2, \frac{4}{3}\right)}$$