MTH 121 — Fall — 2004

**Essex County College — Division of Mathematics**

Test # 1[1] — Created October 26, 2004

**Name:** ________________________________

**Signature:** ________________________________

Show all work *clearly* and in *order*, and box your final answers. Justify your answers algebraically whenever possible. You have at most 80 minutes to take this 100 point exam, each question is worth 10 points. No cellular phones allowed.

1. Find the derivative, $f'(x)$, of the function using the rules.

$$f(x) = \frac{3+x}{1-3x}$$

Solution:

$$f'(x) = \boxed{\frac{(1-3x)\cdot 1 - (3+x)\cdot(-3)}{(1-3x)^2}}$$

If you decide to simplify:

$$f'(x) = \boxed{\frac{10}{(1-3x)^2}}$$

2. Find the derivative, $f'(x)$, of the function using the definition of the derivative.

$$f(x) = \frac{3+x}{1-3x}$$

Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\frac{3+x+h}{1-3x-3h} - \frac{3+x}{1-3x}}{h} \\ &= \lim_{h\to 0} \frac{(3+x+h)(1-3x) - (3+x)(1-3x-3h)}{h(1-3x-3h)(1-3x)} \\ &= \lim_{h\to 0} \frac{\left(3-9x+x-3x^2+h-3xh\right) - \left(3-9x-9h+x-3x^2-3xh\right)}{h(1-3x-3h)(1-3x)} \\ &= \lim_{h\to 0} \frac{10h}{h(1-3x-3h)(1-3x)} \\ &= \lim_{h\to 0} \frac{10}{(1-3x-3h)(1-3x)} \\ &= \boxed{\frac{10}{(1-3x)^2}} \end{aligned}$$

[1] This document was prepared by Ron Bannon using LaTeX.

3. Find the derivative, $f'(x)$, of the function using the rules.

$$f(x) = \sqrt{\frac{x-1}{x+1}}$$

Solution: First rewrite the function.

$$f(x) = \left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$$

Now take the derivative.

$$f'(x) = \boxed{\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}} \cdot \frac{(x+1)\cdot 1 - (x-1)\cdot 1}{(x+1)^2}}$$

If you decide to simplify.

$$f'(x) = \boxed{\frac{1}{(x+1)^2}\sqrt{\frac{x+1}{x-1}}}$$

4. Find the derivative, $f'(x)$, of the function using the rules.

$$f(x) = \sin^2\left(\cos x^2\right)$$

Solution: First rewrite the function.

$$f(x) = \left(\sin\left(\cos x^2\right)\right)^2$$

Now take the derivative.

$$f'(x) = \boxed{2\sin\left(\cos x^2\right) \cdot \cos\left(\cos x^2\right) \cdot \left(-\sin x^2\right) \cdot 2x}$$

If you decide to simplify.

$$f'(x) = \boxed{-4x\sin\left(\cos x^2\right) \cdot \cos\left(\cos x^2\right) \cdot \sin x^2}$$

5. Find the derivative, $\frac{dy}{dx}$, of the relationship using the rules.

$$x^3 + x^2y - 4y^2 = 6$$

Solution: Requires implicit differentiation.

$$\begin{aligned} 3x^2 + x^2y' + 2xy - 8yy' &= 0 \\ 3x^2 + 2xy &= 8yy' - x^2y' \\ 3x^2 + 2xy &= y'\left(8y - x^2\right) \\ \boxed{\frac{3x^2 + 2xy}{8y - x^2}} &= y' = \frac{dy}{dx} \end{aligned}$$

6. Find the domain: $f(x) = \sqrt{4 - 25x^2}$

Solution: Requires solving an inequality.

$$\begin{aligned} 4 - 25x^2 &\geq 0 \\ (2 - 5x)(2 + 5x) &\geq 0 \end{aligned}$$

List the zeros on the number line and perform sign analysis, which gives domain $\boxed{\left[-\frac{2}{5}, \frac{2}{5}\right]}$.

7. Determine if $f(x)$ is even, odd, or neither: $f(x) = 2x^5 - 3x^3 + 2$

Solution: Requires finding:

$$\begin{aligned} f(-x) &= 2(-x)^5 - 3(-x)^3 + 2 = -2x^5 + 3x^3 + 2 \\ -f(x) &= -\left(2x^5 - 3x^3 + 2\right) = -2x^5 + 3x^3 - 2 \\ -f(-x) &= 2x^5 - 3x^3 - 2 \end{aligned}$$

Since $f(x) \neq f(-x)$ we know $f(x)$ is not even. Since $f(x) \neq -f(-x)$ we know $f(x)$ is not odd. Therefore, $f(x)$ is $\boxed{\text{neither}}$.

8. Find, it's either a number, DNE, $+\infty$ or $-\infty$: $\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x} =$

Solution:

$$\begin{aligned} \lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x} &= \lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x} \cdot \frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2} \\ &= \lim_{x \to 0} \frac{x + 4 - 4}{x\left(\sqrt{x+4} + 2\right)} \\ &= \lim_{x \to 0} \frac{1}{\left(\sqrt{x+4} + 2\right)} \\ &= \frac{1}{\left(\sqrt{4} + 2\right)} = \boxed{\frac{1}{4}} \end{aligned}$$

9. Using your calculator, estimate the limit by constructing a table of values. $\lim_{x \to 0} \frac{9^x - 5^x}{x} =$

Solution:

$$\begin{aligned} f(-0.1) &\doteq 0.4860 \\ f(-0.01) &\doteq 0.5767 \\ f(-0.001) &\doteq 0.5867 \\ \lim_{x \to 0} \frac{9^x - 5^x}{x} &\approx \boxed{0.5878} \\ f(0.001) &\doteq 0.5889 \\ f(0.01) &\doteq 0.5991 \\ f(0.1) &\doteq 0.7111 \end{aligned}$$

Therefore, an estimate might be:

$$\lim_{x \to 0} \frac{9^x - 5^x}{x} \approx \boxed{0.5878}$$

Actually, you'll learn how to get the true value of this limit in MTH-122.

$$\lim_{x \to 0} \frac{9^x - 5^x}{x} = \log_e 9 - \log_e 5$$

10. Draw the derivative of $f(x)$ directly below the function.

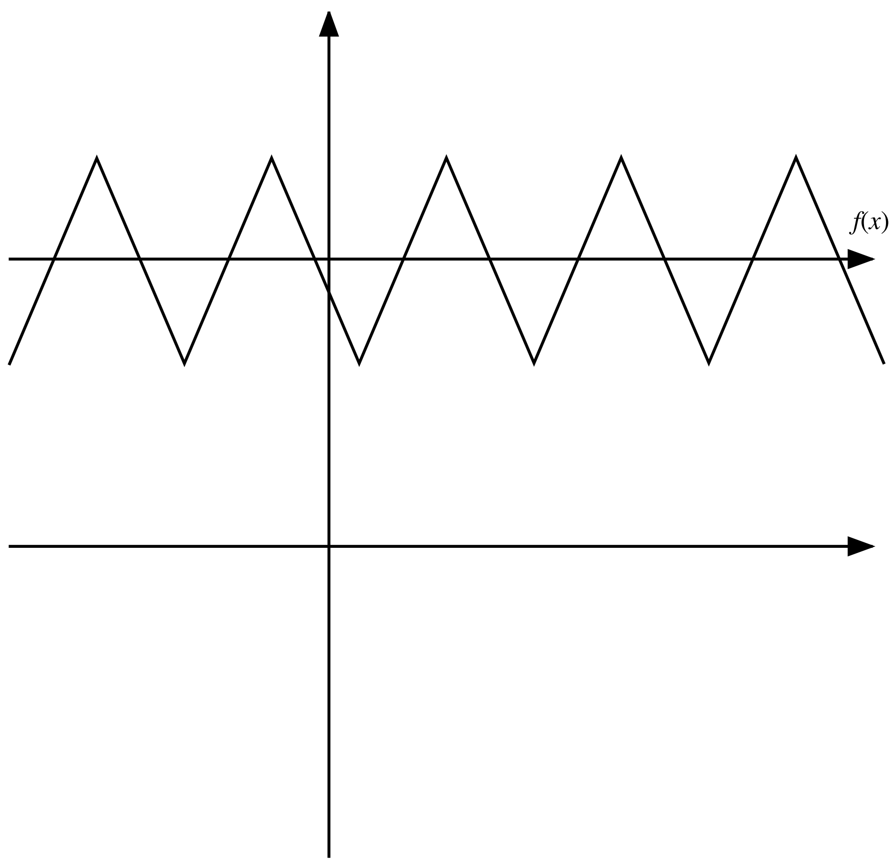


Figure 1: Graph of $f(x)$ given, please graph of $f'(x)$ directly below.

Solution will be discussed in class.