MTH 121 — Fall — 2004
Essex County College — Division of Mathematics
Test # 2[1] — Created December 6, 2004

Name: _____

Signature: _____

Show all work *clearly* and in *order*, and ⎡box your final answers.⎤ Justify your answers algebraically whenever possible. You have at most 80 minutes to take this 100 point exam. No cellular phones allowed.

1. (10 points) — Find the absolute maximum and minimum values of the function $f(x) = x^3 - 3x^2 + 1$ on the interval $-\frac{1}{2} \le x \le 4$.

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2. (15 points) — Given $f(x) = x^2 + 5x + 9$, verify the following:

$$\lim_{n \to \infty} \sum_{i=1}^{n} \left[\frac{8}{n} \cdot f\left(-3 + \frac{8i}{n}\right) \right] = \frac{488}{3}$$

3. (15 points) — A cylindrical can is to be made to hold one liter of oil. Find the dimension that will minimize the cost of the metal to manufacture the can.[2]

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[2]You'll need to find the radius and height. Formulas that might be helpful: volume of a cylinder is given by $V = \pi r^2 h$; circumference of a circle is given by $C = 2\pi r$; area of a circle is given by $A_c = \pi r^2$; and area of a rectangle is given by $A_r = lw$. You should also be aware that $1 \text{ L} = 1{,}000 \text{ cm}^3$.

4. (10 points) — Find $f(x)$ if $f''(x) = 2 + \cos x$, $f(0) = -1$, and $f\left(\frac{\pi}{2}\right) = 0$.

5. (50 points total) — Given:

$$f(x) = \frac{x^2 + 7x + 3}{x^2} = 1 + \frac{7}{x} + \frac{3}{x^2}$$

$$f'(x) = -\frac{7x + 6}{x^3}$$

$$f''(x) = \frac{14x + 18}{x^4}$$

Answer the following questions.

(a) (6 points) — x-intercept(s):

(b) (3 points) — y-intercept(s):

(c) (3 points) — vertical asymptote(s):

(d) (4 points) — horizontal asymptote(s):

(e) (4 points) — domain:

(f) (5 points) — range:

(g) (4 points) — local maximum(s):

(h) (5 points) — local minimum(s):

(i) (4 points) — global maximum(s):

(j) (5 points) — global minimum(s):

(k) (7 points) — point(s) of inflection: