MTH 121 — Fall — 2004

Essex County College — Division of Mathematics

Test # 3[1] — Created December 9, 2004

Name: ________________________________

Signature: ________________________________

Show all work *clearly* and in *order*, and box your final answers. Justify your answers algebraically whenever possible. You have at most 80 minutes to take this 100 point exam. No cellular phones allowed.

1. (10 points) — Find the equations of the tangent line to the curve at the given point.

$$y = \sqrt{2x+1} \qquad (4,3)$$

Solution: First find the derivative.

$$f(x) = y = (2x+1)^{\frac{1}{2}}$$
$$f'(x) = y' = \frac{1}{2}(2x+1)^{-\frac{1}{2}}2 = \frac{1}{\sqrt{2x+1}}$$

Now evaluate the derivative at the point $(4,3)$.

$$f'(4) = \frac{1}{\sqrt{2(4)+1}} = \frac{1}{3}$$

Using the point-slope form, the equation of the tangent line is $\boxed{y - 3 = \frac{1}{3}(x-4)}$. If you used the slope-intercept form, the equation of the tangent line is $\boxed{y = \frac{1}{3}x + \frac{5}{3}}$.

2. (10 points) — Find $f'(x)$ by using the definition.

$$f(x) = \frac{2x+1}{x+3}$$

[1] This document was prepared by Ron Bannon using LaTeX.

Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{2(x+h)+1}{x+h+3} - \frac{2x+1}{x+3}}{h} \\ &= \lim_{h\to 0} \frac{(2x+2h+1)(x+3)-(2x+1)(x+h+3)}{h(x+h+3)(x+3)} \\ &= \lim_{h\to 0} \frac{5h}{h(x+h+3)(x+3)} \\ &= \lim_{h\to 0} \frac{5}{(x+h+3)(x+3)} \\ &= \frac{5}{(x+3)^2} \end{aligned}$$

3. (10 points) — Find $f'(x)$.

$$f(x) = \frac{\sqrt{x}}{x+1}$$

Solution:

$$\begin{aligned} f(x) &= \frac{\sqrt{x}}{x+1} = \frac{(x)^{\frac{1}{2}}}{x+1} \\ f'(x) &= \frac{(x+1)\frac{1}{2}x^{-\frac{1}{2}} - x^{\frac{1}{2}}}{(x+1)^2} = \frac{\frac{1}{2}x^{-\frac{1}{2}}(x+1-2x)}{(x+1)^2} = \frac{1-x}{2\sqrt{x}(x+1)^2} = \frac{\sqrt{x}(1-x)}{2x(x+1)^2} \end{aligned}$$

If you decide not to simplify, your answer is:

$$\boxed{f'(x) = \frac{(x+1)\frac{1}{2}x^{-\frac{1}{2}} - x^{\frac{1}{2}}}{(x+1)^2}}$$

If you decide to simplify, your answer is:

$$\boxed{f'(x) = \frac{1-x}{2\sqrt{x}(x+1)^2}}$$

If you decide to simplify and rationalize the denominator, your answer is:

$$\boxed{f'(x) = \frac{\sqrt{x}(1-x)}{2x(x+1)^2}}$$

4. (10 points) — Find Find $f'(x)$.

$$f(x) = \sin\left(x^2 + 2x - 1\right)$$

Solution:

$$\begin{aligned} f'(x) &= \cos\left(x^2 + 2x - 1\right) \cdot (2x + 2) \\ f'(x) &= 2(x+1)\cos\left(x^2 + 2x - 1\right) \end{aligned}$$

Your answer should be equivalent to: $\boxed{f'(x) = 2(x+1)\cos\left(x^2 + 2x - 1\right)}$.

5. (10 points) — Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$$y^5 + 3x^2y^2 + 5x^4 = 12$$

Solution:

$$\begin{aligned} 5y^4y' + 6x^2yy' + 6xy^2 + 20x^3 &= 0 \\ y'\left(5y^4 + 6x^2y\right) &= -\left(6xy^2 + 20x^3\right) \\ y' &= \boxed{-\frac{6xy^2 + 20x^3}{5y^4 + 6x^2y}} \end{aligned}$$

6. (10 points) — Set up an expression for

$$\int_0^{\pi} \sin x \ \mathrm{d}x$$

as a limit of sums. **Do not evlauate.**

Solution:

$$\int_0^{\pi} \sin x \ \mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{\pi}{n} \cdot \sin\left(\frac{\pi}{n} i\right)\right]$$

7. (10 points) — Evaluate.

$$\int_{-1}^{7} \sqrt{4 + 3x} \ \mathrm{d}x$$

Solution: Let $u = 4 + 3x$, where $\mathrm{d}u = 3 \ \mathrm{d}x$.

$$\int_{-1}^{7} \sqrt{4+3x} \ \mathrm{d}x = \frac{1}{3}\int_{1}^{25} u^{\frac{1}{2}} \ \mathrm{d}u = \frac{2}{9}u^{\frac{3}{2}}\Big|_1^{25} = \frac{2}{9}(125 - 1) = \boxed{\frac{248}{9}}$$

8. (10 points) — Evaluate.

$$\int_0^{\sqrt{\frac{\pi}{2}}} x\cos\left(x^2\right)\ \mathrm{d}x$$

Solution: Let $u = x^2$, where $\mathrm{d}u = 2x\ \mathrm{d}x$.

$$\int_0^{\sqrt{\frac{\pi}{2}}} x\cos\left(x^2\right)\ \mathrm{d}x = \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos u\ \mathrm{d}u = \frac{1}{2}\sin u\Big|_0^{\frac{\pi}{2}} = \frac{1}{2}(1-0) = \boxed{\frac{1}{2}}$$

9. (10 points) — Evaluate.

$$\int_{-1}^{1} \frac{1}{(2x-3)^2}\ \mathrm{d}x$$

Solution: Let $u = 2x - 3$, where $\mathrm{d}u = 2\ \mathrm{d}x$.

$$\int_{-1}^{1} \frac{1}{(2x-3)^2}\ \mathrm{d}x = \frac{1}{2}\int_{-5}^{-1} u^{-2}\ \mathrm{d}u = -\frac{1}{2u}\Big|_{-5}^{-1} = \frac{1}{2} - \frac{1}{10} = \boxed{\frac{2}{5}}$$

10. (10 points total) — Given:

$$\begin{aligned} f(x) &= 2\cos x + \sin 2x \\ f'(x) &= -2(2\sin x - 1)(\sin x + 1) \\ f''(x) &= -2\cos x(1 + 4\sin x) \end{aligned}$$

Answer the following questions where $f(x)$ is restricted to the interval $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

(a) (4 points) — range: Answer: $\boxed{\left[-\frac{3\sqrt{3}}{2}, \frac{3\sqrt{3}}{2}\right]}$

(b) (3 points) — global maximum(s): $\boxed{\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right)}$

(c) (3 points) — global minimum(s): $\boxed{\left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{2}\right)}$