Essex County College—Division of Mathematics
$2 \cdot 3 \cdot 5 \cdot 67 = 2010$ A. Shloming Mathematics Prize Examination

This Prize Examination has 20 questions, for a total of 100 points.

Last Name: _____

First Name: _____

Phone or email: _____

Prize Examination Honor Code: The Prize Examination Honor Code is a statement on academic integrity, it articulates reasonable expectations of students and teachers in establishing and maintaining the highest standards in academic work:

1. that they will not give or receive aid in taking this Prize Examination, including the use of notes and electronic devises;
2. that they will not use any communication device while taking this Prize Examination, either in the room or while on a break. If you have a device that rings or vibrates during the contest, *DO NOT ANSWER IT* or look at it. Prior to the Prize Examination you must turn these devices off and store them away from you for the duration of the Prize Examination. Your Prize Examination will be invalidated and no score may be earned if you use any such device while in the Prize Examination room;
3. that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Prize Examination Honor Code;
4. that they will only turn in their Prize Examination if they are able to honestly state "I do hereby affirm, at the close of this Prize Examination, that I had no unlawful knowledge of the questions or answers prior to the contest and that I have neither given nor received assistance in answering any of the questions during this Prize Examination."

Please sign your name below to record that you have reviewed this Prize Examination Honor Code and will abide by these expectations at all times during this Prize Examination.

Signature: _____

If the question has choices, select one answer; if the question is open ended, write your final answer on the line provided. You do not need to show your work and you will not be given partial credit. *Five points* for each correct answer, and there's no penalty for incorrect answers. No calculators are allowed, and the use of cellular phones is strictly forbidden.

1. [5 points] On Monday a store put out 100 watermelons to be sold, and some were sold. On Tuesday the number left over was doubled, and sales were double what they were on Monday. On Wednesday, the number left over was tripled, and sales were triple what they were on Monday, leaving none left over. How many were sold each day?

1. _____

2. [5 points] If $f(x) = \dfrac{x^4 + x^2}{x + 1}$, then $f(i)$, where $i = \sqrt{-1}$, is equal to

   A. $i + 1$ B. 1 C. -1 D. 0 E. $-1 - i$

2. _____

3. [5 points] The Arzadun family is about to embark on an 18000-mile car trip through Europe. The tires on their car are all new, but each is good for only 12000 miles. What is the smallest number of new spare tires they should take along if they want to make the trip without having to buy any new tires along the way?

3. _____

4. [5 points] A circular ferris wheel has a radius of 8 meters and rotates at a rate of 12 degrees per second. At $t = 0$, a seat is at its lowest point, which is two meters above the ground. Determine how high above the ground the seat is at $t = 40$ seconds.

4. _____

5. [5 points] If the points $(1, y_1)$ and $(-1, y_2)$ lie on the graph of $y = ax^2 + bx + c$, and $y_1 - y_2 = -6$, then b equals

   A. -3 B. 0 C. 3 D. $\sqrt{ac}$ E. $\dfrac{a + c}{2}$

5. _____

6. [5 points] A square and a circle have equal perimeters. The ratio of the area of the circle to the area of the square is

   A. $\frac{4}{\pi}$ B. $\frac{\pi}{\sqrt{2}}$ C. $\frac{4}{1}$ D. $\frac{\sqrt{2}}{\pi}$ E. $\frac{\pi}{4}$

6. _____

7. [5 points] A 100-pound watermelon is 95 percent water by weight. It is dehydrated until it is 90 percent water by weight. What is its weight after dehydration.

7. _____

8. [5 points] The diagonal of one square is four times the length of the diagonal of another. How many times larger is the area of the larger square?

8. _____

9. [5 points] Can the sum of two primes be prime?

9. _____

10. [5 points] Today my daughter is 1/3 my age. Five years ago she was one 1/4 my age then. How old is my daughter now?

10. _____

11. [5 points] The volume of a cube is 8 times greater than the volume of another cube. What is the relationship of their surface areas?

11. _____

12. [5 points] Some women and some horses are in a stable. In all, there are 22 heads and 72 legs. How many women and how many horses are in the stable?

12. _____

13. [5 points] The sum of 49 consecutive integers is $7^5 = 16807$. What is the median?

13. _____

14. [5 points] Circle C_1 has a center $(0,\ 2)$ with radius 2, and circle C_2 has a center $(2,\ 0)$ with radius 2. The circles overlap in the first quadrant. What is the area of this overlap?

14. _____

Graphing the circles is encouraged.

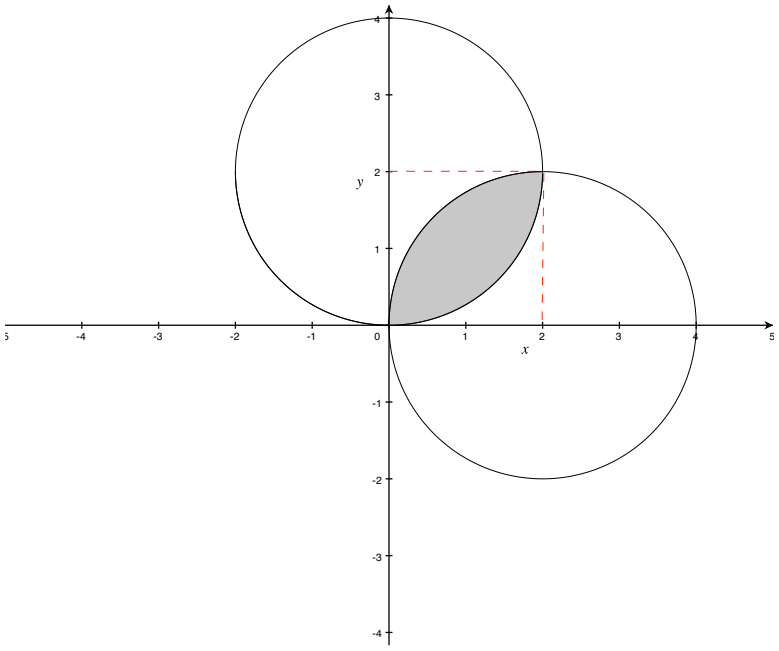

Figure 1: Circles C_1 and C_2

15. [5 points] Two numbers are such that their difference, their sum, and their product are in the ratio $1 : 7 : 24$, respectively. What is their product?

15. _____

16. [5 points] Before going to a garage sale, Elise counted the money she was taking along. After an hour, she counted it again and found that she had spent exactly half of it. The number of cents she now had equaled the number of dollars she started with; the number of dollars she now had was half the number of cents she started with. How much did she spend?

16. _____

17. [5 points] The number of seconds in six weeks equals $n!$. Find n.

17. _____

18. [5 points] For how many positive integers n is $n^3 - 8n^2 + 20n - 13$ prime?

18. _____

19. [5 points] What is the slope of the tangent line to $y = x \sin x$ at $x = \pi$.
A. $-\pi$ B. π C. 0 D. 1 E. -1

19. _____

20. [5 points] What positive number x satisfy

$$\sqrt[3]{x+9} - \sqrt[3]{x-9} = 3?$$

20. _____

END OF EXAM