

Exercise One: The Basics

A certain math professor will **scream** and may even **cry** if you confuse the Latin terms *id est*, meaning “that is to say” and *exempli gratia*, meaning “for instance.” Compare the following.

I adore polynomials, *e.g.*, $x^4 + x^2 + 1$.

I adore polynomials, *i.e.*, expressions of the form $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$, where a_0, a_1, \dots, a_n are constants and n is a non-negative integer.

Do you lie awake at night wondering what is the smallest positive integer that can be written as the sum of two perfect cubes in two distinct ways? Well, wonder no more:

$$1729 = 10^3 + 9^3 = 12^3 + 1^3$$

We all learned in first grade that $(a + b)^3 \neq a^3 + b^3$, but when we study modular arithmetic we’ll find that the two *are* equal in “mod 3.”

The famous mathematician Euler (rhymes with “boiler” **NOT** with “ruler”) used the geometric series formula $a + ar + ar^2 + ar^3 + \dots = a/(1 - r)$ to conclude the following.

$$1 - 1 + 1 - 1 + 1 - \dots = \frac{1}{2}$$

But we know the formula only applies if $-1 < r < 1$, so this result is not valid.