

### Exercise Ten: Tables

Below is the operation table (or Cayley table) for  $(\mathbb{Z}_4, \cdot_4)$ , that is, the integers *modulo* 4 under multiplication. Ask yourself whether or not this forms a group.

$\cdot_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Ah, truth tables ... good times.

This one demonstrates why “proof by contrapositive” works by showing that the statements  $P \Rightarrow Q$  and  $(\sim Q) \Rightarrow (\sim P)$  have the same truth values.

$P$	$Q$	$P \Rightarrow Q$	$\sim P$	$\sim Q$	$(\sim Q) \Rightarrow (\sim P)$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

TABLE 1. Proof by contrapositive