MTH 120 — Pre Calculus II

Essex County College — Division of Mathematics

Sample Review Questions[1] — Created December 14, 2005

At Essex County College you should be prepared to show all work *clearly* and in *order*, ending your work by $\boxed{\text{boxing the answer.}}$ Furthermore, justify your answers algebraically whenever possible. These questions are for review only, and placement tests are not limited to these problems alone. Solutions and work are provided for each question. Please feel free to email rbannon@mac.com with questions or comments pertaining to this document.

1. Find the rational number representation of the given repeating decimal.

$$1.2\overline{87}$$

Solution: Let $x = 1.2\overline{87}$.

$$\begin{aligned} 10x &= 12.\overline{87} \\ 1000x &= 1287.\overline{87} \\ 1000x - 10x &= 1287.\overline{87} - 12.\overline{87} \\ 990x &= 1275.00\overline{00} \\ x &= \boxed{\frac{1275}{990}} = \boxed{\frac{85}{66}} \end{aligned}$$

2. Use summation notation to write the given sum.

$$3 + 9 + 27 + 81 + \cdots + 729$$

Solution: Here's one possible answer.

$$3 + 9 + 27 + 81 + \cdots + 729 = \boxed{\sum_{n=1}^{6} 3^n}$$

3. Evaluate the given infinite geometric sum.

$$\sum_{n=1}^{\infty} 0.9^n$$

Solution: Here's one possible answer.

$$\sum_{n=1}^{\infty} 0.9^n = 0.9 \cdot \frac{1}{1 - 0.9} = \frac{0.9}{0.1} = \boxed{9}$$

[1] This document was prepared by Ron Bannon using LaTeX 2_ε.

4. Use your calculator to approximate each of the following to three decimal places.

(a) $\arcsin(-0.987) \approx \boxed{-1.409 \quad \text{or} \quad -80.751^\circ}$

(b) $\arctan(-89.456) \approx \boxed{-1.560 \quad \text{or} \quad -89.360^\circ}$

(c) $\arccos(-0.009) \approx \boxed{1.580 \quad \text{or} \quad 90.516^\circ}$

5. Find the exact value of the following.

(a) $\arccos\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{\pi}{4} \quad \text{or} \quad 45^\circ}$

(b) $\arccos\left[\sin\left(\frac{5\pi}{3}\right)\right] = \arccos\left(-\frac{\sqrt{3}}{2}\right) = \boxed{\frac{5\pi}{6} \quad \text{or} \quad 150^\circ}$

6. Given

$$\mathbf{u} = \langle 1,\ -5 \rangle, \qquad \mathbf{v} = \langle 5,\ -3 \rangle, \qquad \mathbf{w} = \langle -3,\ -6 \rangle,$$

find.

(a) $5\mathbf{w} + 3\mathbf{v}$

Solution:

$$5\mathbf{w} + 3\mathbf{v} = 5\langle -3,\ -6 \rangle + 3\langle 5,\ -3 \rangle = \langle -15,\ -30 \rangle + \langle 15,\ -9 \rangle = \boxed{\langle 0,\ -39 \rangle}$$

(b) $\mathbf{w} \cdot \mathbf{v}$

Solution:

$$\mathbf{w} \cdot \mathbf{v} = \langle -3,\ -6 \rangle \cdot \langle 5,\ -3 \rangle = -15 + 18 = \boxed{3}$$

(c) $\|\mathbf{u} - \mathbf{w}\|$

Solution:

$$\|\mathbf{u} - \mathbf{w}\| = \|\langle 4,\ 1 \rangle\| = \sqrt{16+1} = \boxed{\sqrt{17}}$$

(d) The angle between $\mathbf{v}$ and $\mathbf{u}$.

Solution:

$$\begin{aligned} \cos\theta &= \frac{\mathbf{v} \cdot \mathbf{u}}{\|\mathbf{v}\|\,\|\mathbf{u}\|} \\ \cos\theta &= \frac{20}{\sqrt{26}\sqrt{34}} = \frac{10}{\sqrt{221}} \\ \theta &= \boxed{\arccos\left(\frac{10}{\sqrt{221}}\right)} \approx \boxed{47.7263109939^\circ} \quad \text{or} \quad \boxed{0.832981266674} \end{aligned}$$

7. Simplify the factorial expression.

$$\frac{(2n+2)!}{(2n)!}$$

Solution:

$$\frac{(2n+2)!}{(2n)!} = \frac{(2n+2)(2n+1)(2n)!}{(2n)!} = \boxed{(2n+2)(2n+1) = 4n^2+6n+2}$$

8. A ramp 21 feet in length rises to a loading platform that is 5 feet off the ground. Assuming that the ground is level, what is the angle (to the nearest whole degree) between the ramp and the ground?

Solution: You should draw a diagram first.

$$\sin\theta = \frac{5}{21}, \quad \Rightarrow \quad \arcsin\left(\frac{5}{21}\right) \approx \boxed{14^\circ}$$

9. Write an algebraic expression that is equivalent to

$$\sec\left[\arcsin(x-1)\right].$$

Solution: You should draw a triangle first and use the Pythagorean Theorem to determine the missing side.

$$\sec\left[\arcsin(x-1)\right] = \boxed{\frac{1}{\sqrt{2x-x^2}}}$$

10. Find the numerical coefficient of the term whose variable part is x^6y^3 in the expansion of

$$(x-2y)^9.$$

Solution:

$$\binom{9}{6}(x)^6(-2y)^3 = 84\cdot(-8)\,x^6y^3 = -672x^6y^3$$

So, the numerical coefficient is $\boxed{-672}$.

11. Write an expression for the n^{th} term.

$$1,\ \frac{5}{2},\ \frac{25}{6},\ \frac{125}{24},\ \frac{625}{120},\ldots$$

Solution:

$$\boxed{a_n = \frac{5^{n-1}}{n!}}$$

12. Convert each of the following angle measures to radian measure.

(a) $60° =$

Solution:

$$60° = 60° \cdot \left(\frac{\pi}{180°}\right) = \boxed{\frac{\pi}{3}}$$

(b) $90° =$

Solution:

$$90° = 90° \cdot \left(\frac{\pi}{180°}\right) = \boxed{\frac{\pi}{2}}$$

(c) $50° =$

Solution:

$$50° = 50° \cdot \left(\frac{\pi}{180°}\right) = \boxed{\frac{5\pi}{18}}$$

13. Solve for x.

(a) $\sin x + \sqrt{2} = -\sin x$ in the interval $[0,\ 2\pi)$.

Solution:

$$\begin{aligned} \sin x + \sqrt{2} &= -\sin x \\ 2\sin x &= -\sqrt{2} \\ \sin x &= -\frac{\sqrt{2}}{2}. \end{aligned}$$

The reference is 45° and the solutions occur in the third and fourth quadrant, so

$$x = 45° + 180° = 225° = \boxed{\frac{5\pi}{4}} \qquad \text{and} \qquad x = 360° - 45° = 315° = \boxed{\frac{7\pi}{4}}$$

(b) $2\sin^2 x - \sin x - 1 = 0$ in the interval $[0,\ 2\pi)$.

Solution:

$$\begin{aligned} 2\sin^2 x - \sin x - 1 &= 0 \\ (2\sin x + 1)(\sin x - 1) &= 0. \end{aligned}$$

Which gives two equations to solve.

$$\sin x = -\frac{1}{2} \qquad \text{and} \qquad \sin x = 1.$$

The reference for the first equation is $30°$ and the solutions occur in the third and fourth quadrant, so

$$x = 30° + 180° = 210° = \boxed{\frac{7\pi}{6}} \qquad \text{and} \qquad x = 360° - 30° = 330° = \boxed{\frac{11\pi}{6}},$$

and the solution to the second equation is $\boxed{x = \frac{\pi}{2}}$

14. Use the Binomial Theorem to expand and simplify

$$\left(3\sqrt[3]{x^2} - 2\sqrt[3]{y}\right)^3.$$

Solution: First the expansion

$$\boxed{\binom{3}{3}\left(3\sqrt[3]{x^2}\right)^3 + \binom{3}{2}\left(3\sqrt[3]{x^2}\right)^2(-2\sqrt[3]{y}) + \binom{3}{1}\left(3\sqrt[3]{x^2}\right)(-2\sqrt[3]{y})^2 + \binom{3}{0}(-2\sqrt[3]{y})^3},$$

then the simplification

$$\boxed{27x^2 - 54\sqrt[3]{x^4y} + 36\sqrt[3]{x^2y^2} - 8y}.$$

15. Evaluate (exact values) all six trigonometric functions for $x = -120°$.

(a) $\sin x$

Solution: $\sin x = \boxed{-\frac{\sqrt{3}}{2}}$

(b) $\cos x$

Solution: $\cos x = \boxed{-\frac{1}{2}}$

(c) $\tan x$

Solution: $\tan x = \boxed{\sqrt{3}}$

(d) $\cot x$

Solution: $\cot x = \boxed{\frac{1}{\sqrt{3}}}$

(e) $\sec x$

Solution: $\sec x = \boxed{-2}$

(f) $\csc x$

Solution: $\csc x = \boxed{-\frac{2}{\sqrt{3}}}$

16. Find the sum.

$$\sum_{n=48}^{70} (2n-1)$$

Solution: Expand first. You should also notice that there are $70 - 48 + 1 = 23$ terms.

$$\begin{aligned}\sum_{n=48}^{70} (2n-1) &= (2\cdot 48 - 1) + (2\cdot 49 - 1) + (2\cdot 50 - 1) + \cdots + (2\cdot 70 - 1)\\ &= 2\cdot(48 + 49 + 50 + \cdots + 70) - (1 + 1 + 1 + \cdots + 1)\\ &= 2\cdot\frac{23}{2}\cdot(70 + 48) - 23 = \boxed{2691}\end{aligned}$$

17. Use mathematical induction to prove the formula for every positive integer n.

$$2\left(1 + 3 + 3^2 + 3^3 + \cdots + 3^{n-1}\right) = 3^n - 1$$

Solution: Verify for $n = 1$.

$$\begin{aligned}P_1 : \quad 2\,(1) &= 3 - 1\\ 2 &= 2\end{aligned}$$

Assume P_k and show that $P_k \to P_{k+1}$.

$$P_k : \quad 2\left(1 + 3 + 3^2 + 3^3 + \cdots + 3^{k-1}\right) = 3^k - 1$$

Add $2 \cdot 3^k$ to both sides.

$$\begin{aligned}P_k : \quad 2\left(1 + 3 + 3^2 + 3^3 + \cdots + 3^{k-1}\right) &= 3^k - 1\\ 2\left(1 + 3 + 3^2 + 3^3 + \cdots + 3^{k-1}\right) + 2\cdot 3^k &= 3^k - 1 + 2\cdot 3^k\\ 2\left(1 + 3 + 3^2 + 3^3 + \cdots + 3^{k-1} + 3^k\right) &= 3\cdot 3^k - 1\\ 2\left(1 + 3 + 3^2 + 3^3 + \cdots + 3^{k-1} + 3^k\right) &= 3^{k+1} - 1\end{aligned}$$

This last line is exactly what we wanted, P_{k+1}. *Q.E.D.*

18. When an airplane leaves the runway, its angle of climb is 19° and its speed is 300 feet per second. Find the plane's altitude after 30 seconds.

Solution: The plane will travel $30 \cdot 300 = 9{,}000$ feet (this is the hypotenuse of a right triangle), so the plane's altitude (this is the side opposite the 19° angle) is $9000 \cdot \sin 19^\circ \approx \boxed{2930}$ feet.

19. Find the first five terms of the recursively defined sequence.

$$a_1 = 15, \quad a_{k+1} = 3a_k - 2.$$

Solution:

$$a_1 = \boxed{15},\ a_2 = 3a_1 - 2 = 3 \cdot 15 - 2 = \boxed{43},\ a_3 = 3a_2 - 2 = 3 \cdot 43 - 2 = \boxed{127},$$
$$a_4 = 3a_3 - 2 = 3 \cdot 127 - 2 = \boxed{379},\ a_5 = 3a_4 - 2 = 3 \cdot 379 - 2 = \boxed{1135}.$$

20. Use your calculator to evaluate the trigonometric function. Round your answers to five decimal places.

(a) $\sin 11.67°$

Solution: $\sin 11.67° \approx \boxed{0.20227}$

(b) $\cos 0.345$

Solution: $\cos 0.345 \approx \boxed{0.94108}$

(c) $\tan\left(-\frac{8\pi}{9}\right)$

Solution: $\tan\left(-\frac{8\pi}{9}\right) \approx \boxed{0.36397}$

(d) $\cot 2.379$

Solution: $\cot 2.379 = \frac{1}{\tan 2.379} \approx \boxed{-1.04668}$

(e) $\csc(-2.689)$

Solution: $\csc(-2.689) = \frac{1}{\sin(-2.689)} \approx \boxed{-2.28677}$

(f) $\sec 45$

Solution: $\sec 45 = \frac{1}{\cos 45} \approx \boxed{1.90359}$

(g) $\arcsin 0.564$

Solution: $\arcsin 0.564 \approx \boxed{0.59922}$ or $\arcsin 0.564 \approx \boxed{34.33288°}$

(h) $\arccos(-0.367)$

Solution: $\arccos(-0.367) \approx \boxed{1.94658}$ or $\arccos(-0.367) \approx \boxed{111.53072°}$

(i) $\arctan 1.113$

Solution: $\arctan 1.113 \approx \boxed{0.83883}$ or $\arctan 1.113 \approx \boxed{48.06117°}$

21. Find the infinite geometric sum.

$$8 + 6 + \frac{9}{2} + \frac{27}{8} + \cdots$$

Solution: $a_1 = 8$ and $r = \frac{6}{8} = \frac{3}{4}$.

$$S_\infty = 8 \cdot \frac{1}{1 - \frac{3}{4}} = \boxed{32}$$

22. Let

$$a_n = n2^n$$

(a) Write the first five terms of the sequence, starting with a_1.

Solution:

$a_1 = 1{\cdot}2^1 = \boxed{2}$, $a_2 = 2{\cdot}2^2 = \boxed{8}$, $a_3 = 3{\cdot}2^3 = \boxed{24}$, $a_4 = 4{\cdot}2^4 = \boxed{64}$, $a_5 = 5{\cdot}2^5 = \boxed{160}$

(b) Is this sequence arithmetic, geometric, or neither.

Solution: $\boxed{\text{Neither}}$.

23. Find θ in degrees ($0° < \theta < 90°$) and radians ($0 < \theta < \frac{\pi}{2}$), if

$$\cot\theta = \frac{1}{\sqrt{3}}.$$

Solution: Draw a triangle first, clearly $\theta = \boxed{60° = \frac{\pi}{3}}$.

24. Find the sum.

$$\sum_{n=51}^{100} 6n$$

Solution: Expand first.

$$\begin{aligned}\sum_{n=51}^{100} 6n &= 6 \cdot 51 + 6 \cdot 52 + 6 \cdot 53 + \cdots + 6 \cdot 100 \\ &= 6 \cdot (51 + 52 + 53 + \cdots + 100) \\ &= 6 \cdot \frac{50}{2} \cdot (100 + 51) = \boxed{22650}\end{aligned}$$

25. Use the properties of inverse functions to find the exact value of

$$\arccos\left(\cos\frac{7\pi}{2}\right).$$

Solution:

$$\arccos\left(\cos\frac{7\pi}{2}\right) = \arccos(0) = \boxed{\frac{\pi}{2}}$$

26. Sketch one period of the graph of the function.

$$f(x) = \sin(\pi x + 2\pi) + 1$$

Solution: I'm mainly looking for five points.

$$\boxed{(-2,\ 1),\quad \left(-\frac{3}{2},\ 2\right),\quad (-1,\ 1),\quad \left(-\frac{1}{2},\ 0\right),\quad (0,\ 1)}.$$

They should be plotted and then connected using a sine wave.

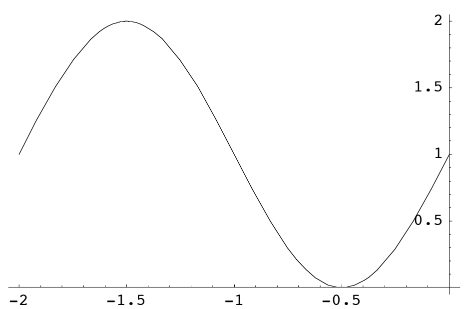

Figure 1: Graph of $f(x) = \sin(\pi x + 2\pi) + 1$.

27. A deposit of P dollars is made at the beginning of each month in an account earning an annual interest rate r, compounded monthly. The balance after t years is given by

$$A = \sum_{i=1}^{12t} P\left(1 + \frac{r}{12}\right)^i.$$

Show that A is also equal to

$$A = P\left[\left(1 + \frac{r}{12}\right)^{12t} - 1\right]\left(1 + \frac{12}{r}\right).$$

Solution: Expand $\sum_{i=1}^{12t} P\left(1+\frac{r}{12}\right)^i$ and you should notice that it is geometric. Just simplify to show that it's the same as $A = P\left[\left(1+\frac{r}{12}\right)^{12t} - 1\right]\left(1+\frac{12}{r}\right)$.

$$\begin{aligned}
A &= P\left(1+\frac{r}{12}\right)\cdot\left[1+\left(1+\frac{r}{12}\right)+\left(1+\frac{r}{12}\right)^2+\left(1+\frac{r}{12}\right)^3+\cdots+\left(1+\frac{r}{12}\right)^{12t-1}\right] \\
&= P\left(1+\frac{r}{12}\right)\cdot\frac{1-\left(1+\frac{r}{12}\right)^{12t}}{1-\left(1+\frac{r}{12}\right)} \\
&= P\left(1+\frac{r}{12}\right)\cdot\frac{1-\left(1+\frac{r}{12}\right)^{12t}}{-\frac{r}{12}} \\
&= P\left(1+\frac{r}{12}\right)\cdot\left[1-\left(1+\frac{r}{12}\right)^{12t}\right]\cdot\left(-\frac{12}{r}\right) \\
&= P\left(1+\frac{r}{12}\right)\cdot\left(\frac{12}{r}\right)\cdot\left[1-\left(1+\frac{r}{12}\right)^{12t}\right]\cdot(-1) \\
&= P\left(\frac{12}{r}+1\right)\cdot\left[-1+\left(1+\frac{r}{12}\right)^{12t}\right] \\
&= P\left[\left(1+\frac{r}{12}\right)^{12t}-1\right]\left(1+\frac{12}{r}\right)
\end{aligned}$$

Q.E.D.

28. Use $\csc\theta = 3$ and $\sec\theta = \frac{3\sqrt{2}}{4}$ to find the exact value of each of the following.

(a) The quadrant that θ is in.

Solution: $\boxed{\text{First (I) quadrant}}$.

(b) $\sin\theta$

Solution: $\sin\theta = \boxed{\frac{1}{3}}$

(c) $\tan\theta$

Solution: $\tan\theta = \boxed{\frac{\sqrt{2}}{4}}$

(d) $\cos\theta$

Solution: $\cos\theta = \boxed{\frac{4}{3\sqrt{2}}}$

(e) $\sec(90^\circ - \theta)$

Solution: $\sec(90^\circ - \theta) = \boxed{3}$

29. Perform the indicated addition and use the fundamental identities to simplify.

$$\frac{1}{1-\sin x}+\frac{1}{1+\sin x}$$

Solution:

$$\begin{aligned}
\frac{1}{1-\sin x}+\frac{1}{1+\sin x} &= \frac{1}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}+\frac{1}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}\\
&= \frac{1+\sin x}{1-\sin^2 x}+\frac{1-\sin x}{1-\sin^2 x}\\
&= \frac{1+\sin x+1-\sin x}{1-\sin^2 x}\\
&= \frac{2}{1-\sin^2 x}\\
&= \boxed{\frac{2}{\cos^2 x}} = \boxed{2\sec^2 x}
\end{aligned}$$

30. Write an expression for the n^{th} term.

$$2,\ 1,\ \frac{8}{9},\ 1,\ \frac{32}{25},\ldots$$

Solution: By inspection.

$$\boxed{a_n = \frac{2^n}{n^2}}$$

31. If

$$\sin\alpha = -\frac{5}{13} \qquad \text{and} \qquad \cos\beta = \frac{3}{5},$$

with both α and β are in the fourth quadrant. Find the exact values of the following.

Preamble: Since α is in the fourth quadrant $\frac{\alpha}{2}$ will be in the second quadrant, which will determine the signs of the half-angle formulas.

$$\begin{aligned}
\sin\alpha = -\frac{5}{13} &\qquad \text{and} \qquad \cos\alpha = \frac{12}{13}\\
\sin\beta = -\frac{4}{5} &\qquad \text{and} \qquad \cos\beta = \frac{3}{5}
\end{aligned}$$

(a) $\sin(\alpha-\beta)$

Solution:

$$\begin{aligned}
\sin(\alpha-\beta) &= \sin\alpha\ \cos\beta - \sin\beta\ \cos\alpha\\
&= \left(-\frac{5}{13}\right)\left(\frac{3}{5}\right) - \left(-\frac{4}{5}\right)\left(\frac{12}{13}\right)\\
&= \boxed{\frac{33}{65}}
\end{aligned}$$

(b) $\cos(\alpha - \beta)$

Solution:

$$\begin{aligned} \cos(\alpha - \beta) &= \cos\alpha \ \cos\beta + \sin\alpha \ \sin\beta \\ &= \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) + \left(-\frac{5}{13}\right)\left(-\frac{4}{5}\right) \\ &= \boxed{\frac{56}{65}} \end{aligned}$$

(c) $\tan(\alpha - \beta)$

Solution:

$$\begin{aligned} \tan(\alpha - \beta) &= \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} \\ &= \frac{\frac{33}{65}}{\frac{56}{65}} \\ &= \boxed{\frac{33}{56}} \end{aligned}$$

(d) $\sin\left(\frac{\alpha}{2}\right)$

Solution: $\frac{\alpha}{2}$ is in the second quadrant, so the sine will be positive.

$$\begin{aligned} \sin\left(\frac{\alpha}{2}\right) &= \sqrt{\frac{1 - \cos\alpha}{2}} \\ &= \sqrt{\frac{1 - 12/13}{2}} \\ &= \boxed{\frac{1}{\sqrt{26}}} \end{aligned}$$

(e) $\cos\left(\frac{\alpha}{2}\right)$

Solution:$\frac{\alpha}{2}$ is in the second quadrant, so the cosine will be negative.

$$\begin{aligned} \cos\left(\frac{\alpha}{2}\right) &= -\sqrt{\frac{1 + \cos\alpha}{2}} \\ &= -\sqrt{\frac{1 + 12/13}{2}} \\ &= \boxed{-\frac{5}{\sqrt{26}}} \end{aligned}$$

(f) $\tan\left(\frac{\alpha}{2}\right)$

Solution: Just take the ratio of sine over cosine.

$$\begin{aligned}\tan\left(\frac{\alpha}{2}\right) &= \frac{\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)} \\ &= \boxed{-\frac{1}{5}}\end{aligned}$$

32. Find all solutions.

$$2\sin^2 x + 3\cos x - 3 = 0$$

Solution: You'll need to write it in terms of cosines first.

$$\begin{aligned}2\sin^2 x + 3\cos x - 3 &= 0 \\ 2\left(1-\cos^2 x\right) + 3\cos x - 3 &= 0 \\ -2\cos^2 x + 3\cos x - 1 &= 0\end{aligned}$$

Now multiply both sides of this equation by -1 and factor.

$$\begin{aligned}2\cos^2 x - 3\cos x + 1 &= 0 \\ (2\cos x - 1)(\cos x - 1) &= 0\end{aligned}$$

Set each factor equal to zero and solve.

$$\begin{aligned}\cos x = \frac{1}{2} \quad &\Rightarrow \quad x = \boxed{\frac{\pi}{3} + 2\pi k, \quad k \in \mathbb{Z}} \\ & = \boxed{\frac{5\pi}{3} + 2\pi k, \quad k \in \mathbb{Z}} \\ \cos x = 1 \quad &\Rightarrow \quad x = \boxed{2\pi k, \quad k \in \mathbb{Z}}.\end{aligned}$$

33. If $z = -1 - 1i$, find the trigonometric form of z and z^9, also find z^9 in standard form.

Solution: Here $\theta = 225°$ or $\theta = \frac{5\pi}{4}$, and $r = \sqrt{2}$.

$$\begin{aligned}z &= \boxed{\sqrt{2}\cos 225° + \left(\sqrt{2}\sin 225°\right) i} \\ z^9 &= \boxed{16\sqrt{2}\cos 2025° + \left(16\sqrt{2}\sin 2025°\right) i} \\ z^9 &= \boxed{-16 - 16i}\end{aligned}$$

34. Verify the identity.

$$\sqrt{\frac{1+\sin x}{1-\sin x}} = \frac{1+\sin x}{|\cos x|}$$

Solution: Select the left side.

$$\begin{aligned}
\sqrt{\frac{1+\sin x}{1-\sin x}} &= \sqrt{\frac{1+\sin x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}} \\
&= \sqrt{\frac{(1+\sin x)^2}{1-\sin^2 x}} \\
&= \sqrt{\frac{(1+\sin x)^2}{\cos^2 x}} \\
&= \frac{1+\sin x}{|\cos x|}
\end{aligned}$$

Q.E.D.[2]

35. Rewrite the expression so that it is not in *fractional* form

$$\frac{\tan^2 x}{\csc x + 1}.$$

Solution:

$$\begin{aligned}
\frac{\tan^2 x}{\csc x + 1} &= \frac{\tan^2 x}{\frac{1}{\sin x}+1}\cdot\frac{\sin x}{\sin x} \\
&= \frac{\tan^2 x\ \sin x}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x} \\
&= \frac{\tan^2 x\ \sin x\cdot(1-\sin x)}{1-\sin^2 x} \\
&= \frac{\tan^2 x\ \sin x\cdot(1-\sin x)}{\cos^2 x} \\
&= \boxed{\tan^2 x\ \sin x\cdot(1-\sin x)\cdot\sec^2 x} \\
&= \boxed{\sin^3 x\cdot(1-\sin x)\cdot\sec^4 x}
\end{aligned}$$

36. Find the sum.

$$\frac{2\pi}{3}+\frac{\pi}{4}+\frac{\pi}{12} =$$

Solution:

$$\frac{2\pi}{3}+\frac{\pi}{4}+\frac{\pi}{12} = \frac{8\pi+3\pi+\pi}{12} = \boxed{\pi}$$

[2]Here it might be nice to mention that $1+\sin x \geq 0$ so $\sqrt{(1+\sin x)^2} = 1+\sin x$. However, since $-1 \leq \cos x \leq 1$, the $\sqrt{\cos^2 x} = |\cos x|$.

37. Find all solutions.

$$2\cos^2 x + 3\sin x - 3 = 0$$

Solution: You'll need to write it in terms of cosines first.

$$\begin{aligned} 2\cos^2 x + 3\sin x - 3 &= 0 \\ 2\left(1 - \sin^2 x\right) + 3\sin x - 3 &= 0 \\ -2\sin^2 x + 3\sin x - 1 &= 0 \end{aligned}$$

Now multiply both sides of this equation by -1 and factor.

$$\begin{aligned} 2\sin^2 x - 3\sin x + 1 &= 0 \\ (2\sin x - 1)(\sin x - 1) &= 0 \end{aligned}$$

Set each factor equal to zero and solve.

$$\begin{aligned} \sin x = \frac{1}{2} \quad &\Rightarrow \quad x = \boxed{\frac{\pi}{6} + 2\pi k, \quad k \in \mathbb{Z}} \\ &\qquad\quad\;\; = \boxed{\frac{5\pi}{6} + 2\pi k, \quad k \in \mathbb{Z}} \\ \sin x = 1 \quad &\Rightarrow \quad x = \boxed{\frac{\pi}{2} + 2\pi k, \quad k \in \mathbb{Z}}. \end{aligned}$$

38. Solve the triangle, given $a = 11.23$ inches $b = 8.24$ inches, and $B = 29.84°$.

Solution: You should, of course, draw a triangle first. This one actually gives two distinct answers. I'll give you credit for either one.

$\boxed{A = 42.70°,\ C = 107.46° \text{ and } c = 15.80 \text{ inches.}}$

or

$\boxed{A = 137.30°,\ C = 12.86° \text{ and } c = 3.69 \text{ inches.}}$

39. Solve the triangle, given $a = 13.25$ inches $b = 7.28$ inches, and $c = 9.02$ inches.

Solution: You should, of course, draw a triangle first.

$\boxed{A = 108.28°,\ B = 31.45° \text{ and } C = 40.27°.}$

40. Determine two coterminal angles in radian measure (one positive, one negative) for $\theta = \dfrac{\pi}{3}$.

Solution:

$$\frac{\pi}{3} + 2\pi = \boxed{\frac{7\pi}{3}} \quad \text{and} \quad \frac{\pi}{3} - 2\pi = \boxed{-\frac{5\pi}{3}}$$

41. Find the supplement of 83°.

Solution:

$$180° - 83° = \boxed{97°}$$

42. Find the complement of $\frac{\pi}{6}$.

Solution:

$$\frac{\pi}{2} - \frac{\pi}{6} = \boxed{\frac{\pi}{3}}$$

43. Find the length of arc on a circle of radius 14 inches and a central angle of 60°.

Solution:

$$S = 14 \cdot \frac{\pi}{3}$$

So, the length is $\boxed{\frac{14\pi}{3} \text{ inches}}$.

44. If the $\sec\theta = 7$ and $270° < \theta < 360°$, find the following.

Solution: From the information given we can conclude that $x = 1$, $r = 7$, and that $y < 0$. To find the value of y, solve

$$7^2 = 1^2 + y^2 \quad \Rightarrow \quad y = \pm\sqrt{48} = \pm 4\sqrt{3}.$$

So, $y = -4\sqrt{3}$

(a) $\sin\theta = \boxed{\frac{-4\sqrt{3}}{7}}$

(b) $\cos\theta = \boxed{\frac{1}{7}}$

(c) $\tan\theta = \boxed{-4\sqrt{3}}$

(d) $\cot\theta = \boxed{\frac{-1}{4\sqrt{3}}}$

(e) $\csc\theta = \boxed{\frac{-7}{4\sqrt{3}}}$

45. Given that

$$P_k = \frac{k}{2} \cdot [5k - 3],$$

find P_{k+1}.

Solution: Just replace k with $(k + 1)$.

$$\boxed{P_{k+1} = \frac{(k+1)}{2} \cdot [5(k+1) - 3]}.$$

46. Verify the identity.

$$\sin^2\theta = \frac{\sec^2\theta - 1}{\sec^2\theta}$$

Solution: Select the right side.

$$\begin{aligned}\frac{\sec^2\theta - 1}{\sec^2\theta} &= \frac{\sec^2\theta}{\sec^2\theta} - \frac{1}{\sec^2\theta}\\ &= 1 - \cos^2\theta\\ &= \sin^2\theta\end{aligned}$$

Q.E.D.