MTH 122 — Calculus II

Essex County College — Division of Mathematics and Physics[1]

Lecture Notes #6 — Sakai Web Project Material

1 Select Solutions

1. Show.[2]

$$\int \tan x \, \mathrm{d}x = \ln|\sec x| + C$$

Work: This was done in class.

2. Show.[3]

$$\int \sec x \, \mathrm{d}x = \ln|\sec x + \tan x| + C$$

Work: This was done in class.

3. Show.[4]

$$\int \csc x \, \mathrm{d}x = \ln|\csc x - \cot x| + C$$

Work: This was done in class.

Yes, I believe that above problems are pretty straight forward. Everyone should be able to do these—but please don't try to memorize them. Reminds me of how a good game player works, they don't memorize moves, but they instead recognize situations. So I think if you take the hints as given—but feel free to try your own suggestions—you'll at least move forward, albeit even if you perpetually get stuck! Getting stuck means you're at least trying!

Okay, please try your *level best* before looking at my work on the next page.

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[1] This document was prepared by Ron Bannon (`ron.bannon@mathography.org`) using LaTeX 2ε. Last revised January 10, 2009.

[2] Hint: rewrite tangent as a ratio, and then use u-substitution. We did this before, but I think many will not recall this. Although nice to know, it is not required knowledge.

[3] Hint: rewrite integrand, multiplying it by the unit factor

$$1 = \frac{\sec x + \tan x}{\sec x + \tan x},$$

then u-substitution, with $u = \sec x + \tan x$.

[4] Hint: rewrite integrand, multiplying it by the unit factor

$$1 = \frac{\csc x - \cot x}{\csc x - \cot x},$$

then u-substitution, with $u = \csc x - \cot x$.

4. Integrate.[5]

$$\int \frac{\tan^2 x}{\cos x}\ \mathrm{d}x$$

Work: This may be a tough one, but let's proceed as suggested.

$$\begin{aligned}\int \frac{\tan^2 x}{\cos x}\ \mathrm{d}x &= \int \tan^2 x\ \sec x\ \mathrm{d}x \\ &= \int \tan x\ \tan x\ \sec x\ \mathrm{d}x\end{aligned}$$

Now, as suggested I will try integration by parts, where:

$$u = \tan x \qquad \Rightarrow \qquad \mathrm{d}u = \sec^2 x\ \mathrm{d}x,$$

and

$$\mathrm{d}v = \tan x\ \sec x\ \mathrm{d}x \qquad \Rightarrow \qquad v = \sec x.$$

Useing these parts, we have.

$$\begin{aligned}\int \tan x\ \tan x\ \sec x\ \mathrm{d}x &= \tan x\ \sec x - \int \sec^3 x\ \mathrm{d}x \\ &= \tan x\ \sec x - \int \sec x\ \sec^2 x\ \mathrm{d}x \\ &= \tan x\ \sec x - \int \sec x\ \left(1 + \tan^2 x\right)\ \mathrm{d}x \\ &= \tan x\ \sec x - \int \sec x + \sec x\ \tan^2 x\ \mathrm{d}x \\ &= \tan x\ \sec x - \int \sec x\ \mathrm{d}x - \int \sec x\ \tan^2 x\ \mathrm{d}x \\ \int \tan^2 x\ \sec x\ \mathrm{d}x &= \tan x\ \sec x - \int \sec x\ \mathrm{d}x - \int \tan^2 x\ \sec x\ \mathrm{d}x \\ 2\int \tan^2 x\ \sec x\ \mathrm{d}x &= \tan x\ \sec x - \int \sec x\ \mathrm{d}x \\ \int \tan^2 x\ \sec x\ \mathrm{d}x &= \boxed{\frac{\tan x\ \sec x - \ln\left|\sec x + \tan x\right|}{2} + C}\end{aligned}$$

Okay, this was tough, but you should nonetheless be able to follow my work.

5. Integrate.[6]

$$\int \frac{\cos x + \sin x}{\sin 2x}\ \mathrm{d}x$$

Work: This was done in class.

[5] Hint: rewrite in terms of tangents and secants. You'll need to use problem 2, and you'll need to use integration by parts.

[6] Hint: it's related to problem 2 and 3.

6. Integrate.

$$\int \frac{1}{\cos x - 1}\, \mathrm{d}x$$

Work: No hint given, so this one shouldn't be so *bad*.

$$\begin{aligned}
\int \frac{1}{\cos x - 1}\, \mathrm{d}x &= \int \frac{1}{\cos x - 1} \cdot \frac{\cos x + 1}{\cos x + 1}\, \mathrm{d}x \\
&= \int \frac{\cos x + 1}{\cos^2 x - 1}\, \mathrm{d}x \\
&= -\int \frac{\cos x + 1}{\sin^2 x}\, \mathrm{d}x \\
&= -\int \csc^2 x\, \mathrm{d}x - \int \frac{\cos x}{\sin^2 x}\, \mathrm{d}x \\
&= \cot x - \int \frac{\cos x}{\sin^2 x}\, \mathrm{d}x
\end{aligned}$$

Now use u-substitution on the remaining integral, where:

$$u = \sin x \qquad \Rightarrow \qquad \mathrm{d}u = \cos x\, \mathrm{d}x.$$

So, we have:

$$-\int \frac{\cos x}{\sin^2 x}\, \mathrm{d}x = -\int^{*} \frac{1}{u^2}\, \mathrm{d}u = -\int^{*} u^{-2}\, \mathrm{d}u = \frac{1}{u} + C = \csc x + C.$$

Finally we have:

$$\begin{aligned}
\int \frac{1}{\cos x - 1}\, \mathrm{d}x &= \cot x - \int \frac{\cos x}{\sin^2 x}\, \mathrm{d}x \\
&= \boxed{\cot x + \csc x + C}
\end{aligned}$$

Again, even with hints, you might be lost. However, I want to stress that practice will make these problems seem trivial. The reason for giving hints is to push everyone in the same *reasonable* direction. You should, if resourceful, be able to follow your own lead. Bear in mind, hints are given to guide the practiced mind.

Important: See a mathematical error? If so, please let me know and I'll give you extra credit. Yes errors do occur, and I want to encourage everyone to look carefully at the prepared material.