MTH 122 — Calculus II
Essex County College — Division of Mathematics and Physics[1]
Lecture Notes #14 — Sakai Web Project Material

1 Pre-Calculus Review Problems

1. List the first six terms, starting with $n = 1$, of the sequence whose n^{th} term is given by:

$$a_n = \frac{\cos(n\pi) + n^2}{3 + 2n}$$

2. Find a general term for the sequence:

$$\frac{7}{2}, \frac{7}{5}, \frac{7}{8}, \frac{7}{11}, \frac{1}{2}, \frac{7}{17}, \cdots$$

3. List the first eight terms, starting with $n = 1$, of the sequence whose n^{th} term is given by:

$$a_n = \frac{a_{n-1} + a_{n-2}}{2}, \quad n > 2; \qquad a_1 = 0, \qquad a_2 = 1.$$

4. Give a recursive formula for the sequence:

$$0, 1, 1, 2, 3, 5, \cdots$$

5. Give a recursive formula for the sequence:

$$1, 3, 7, 15, 31, 63, \cdots$$

6. Show that the n^{th} partial sum of a geometric series is

$$S_n = a + ax + ax^2 + ax^3 + ax^4 + \cdots + ax^n = \frac{a\left(1 - x^{n+1}\right)}{1 - x}$$

[1] This document was prepared by Ron Bannon (ron.bannon@mathography.org) using $\LaTeX 2_\varepsilon$. Last revised January 10, 2009.

7. Use this formula to solve the following problem, "Suppose you are offered a 35-hour job that pays \$0.01 the first hour, \$0.02 the second hour, \$0.04 the third hour, and so on. Each hour you work, your earnings are twice your earnings for the previous hour. Exactly *how much money* would you have earned if you worked all thirty-five hours?"

8. If we allow $n \to \infty$, the geometric sum has infinitely many terms. Show that

$$\lim_{n\to\infty} S_n = S = a + ax + ax^2 + ax^3 + ax^4 + \cdots = \frac{a}{1-x},$$

provided $-1 < x < 1$.

9. Use this formula to show that

$$1 = 0.\bar{9}$$

2 Introduction to Sequences and Series

1. **Definition**: A sequence $\{a_n\}$ has a limit L and we write

$$\lim_{n\to\infty} a_n = L$$

if we can make the term a_n as close to L as we like by taking n sufficiently large. If this limit exists, we say the sequence converges, otherwise we say the sequence diverges.

Examples: Does the sequence converge or diverge?

(a)

$$a_n = \frac{1-e^{-n}}{1+e^n}$$

(b)

$$a_n = (-1)^n$$

(c)

$$a_n = (-0.7)^n$$

(d)

$$a_n = 1 + (-1)^n$$

(e)

$$a_n = \frac{1 + (-1)^n}{n}$$

2. **Definition**: A sequence $\{a_n\}$ is called increasing if $a_n < a_{n+1}$ for all $n \geq 1$, that is, $a_1 < a_2 < a_3 < \cdots$. It is called decreasing if $a_n > a_{n+1}$ for all $n \geq 1$. It is called monotonic if it is either increasing of decreasing.

   Examples: Determine if the sequence is increasing, decreasing, or not monotonic.[2]

   (a)

$$a_n = \frac{2n - 3}{3n + 4}$$

   (b)

$$a_n = ne^{-n}$$

   (c)

$$a_n = n + \frac{1}{n}$$

   (d)

$$a_n = \frac{n}{n^2 + 1}$$

[2] A sequence is called monotonic if it is increasing or decreasing.