MTH 122 — Calculus II

Essex County College — Division of Mathematics and Physics[1]

Lecture Notes #17 — Sakai Web Project Material

1 Introduction to Sequences and Series, Part IV

1. **Comparison Test**: Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms.

   (a) If $\sum_{n=1}^{\infty} b_n$ is convergent, and $0 < a_n \le b_n$ for all n, then $\sum_{n=1}^{\infty} a_n$ is also convergent.

   (b) If $\sum_{n=1}^{\infty} b_n$ is divergent, and $a_n \ge b_n > 0$ for all n, then $\sum_{n=1}^{\infty} a_n$ is also divergent.

   Example: Use the comparison test to determine whether $\sum_{n=1}^{\infty} \frac{1}{n^3+1}$ converges.

   Work: For $n \ge 1$ we know that $n^3 \le n^3 + 1$, so

   $$0 < \frac{1}{n^3+1} \le \frac{1}{n^3}.$$

   You should note that

   $$\sum_{n=1}^{\infty} \frac{1}{n^3}$$

   is a convergent p-series. The conclusion, using the **Comparison Test**, is that

   $$\sum_{n=1}^{\infty} \frac{1}{n^3+1}$$

   also $\boxed{\text{converges}}$.

   The two big series that you should use for comparisons are the p-series and the geometric series. The harmonic series is a p-series, with $p = 1$.

   The Geometric Series

   $$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots$$

   is convergent if $|r| < 1$, and its sum is

   $$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots = \frac{a}{1-r}.$$

   If $|r| \ge 1$, the geometric series is divergent.

[1]This document was prepared by Ron Bannon (`ron.bannon@mathography.org`) using $\LaTeX 2_\varepsilon$. Last revised January 10, 2009.

The p-series

$$\sum_{n=1}^{\infty} n^{-p}$$

is convergent if $p > 1$, and is divergent if $p \leq 1$.

1.1 Examples

(a) Use the comparison test to determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$ converges.[2]

(b) Use the comparison test to determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^3+3}$ converges.[3]

(c) Use the comparison test to determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{6n^2+1}{2n^3-1}$ converges.[4]

(d) Use the comparison test to determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n+1}$ converges.[5]

(e) Use the comparison test to determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$ converges.[6]

(f) Use the comparison test to determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^5}}$ converges.[7]

[2] Converges. Use the p-series for comparison.
[3] Converges. Use the p-series for comparison.
[4] Diverges. Use the harmonic series for comparison.
[5] Converges. Use the geometric series for comparison.
[6] Diverges. Use the harmonic series for comparison.
[7] Diverges. Use the p-series for comparison.