MTH 122 — Calculus II

Essex County College — Division of Mathematics and Physics[1]

Project #3 — Sakai Web Project Material

Name: _____

Signature: _____

The following question is worth ten points total, and will be added to your WebAssign grades. Only correct answers will be accepted. Due date will be announce in class.[2]

A student came to me the other day with this question, find the exact value of:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}.$$

I looked at it, and was perplexed.[3] So I decided to use my calculator to find an approximation. I also used Mathematica to see if it knew the answer, surprisingly it did. Then, as I suspected from the get-go, I knew it could be done.

Answer the following questions.

1. Use a computer to calculate (20 decimal places!)

$$\sum_{n=0}^{1000} \frac{(-1)^n}{2n+1}.$$

Solution: I am using Mathematica

$$\sum_{n=0}^{1000} \frac{(-1)^n}{2n+1} = \boxed{0.78564791358488576273}$$

2. Use Mathemtica (*exact value computation*) to calculate

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$$

Solution: I am using Mathematica

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = \boxed{\frac{\pi}{4}}$$

[1]This document was prepared by Ron Bannon (`ron.bannon@mathography.org`) using LaTeX 2ε. Last revised April 7, 2009.

[2]Project questions are assigned on occasion, and have strict due dates that must be adhered to.

[3]Okay, I must be getting old.

3. Look back over your notes to see if I gave you this power series (I did) and then show that Mathematica's results are true.

   Work: On sheet 12, page 5, I gave you

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad -1 \le x \le 1.$$

   Using this with $x = 1$ in this power series, I get.

$$\arctan 1 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = \frac{\pi}{4}$$