Name: _____

Signature: _____

Show all work clearly and in order, and box your final answers. Justify your answers whenever possible. You have 20 minutes to take this 10 point quiz.

The number π is often approximated, and many students believe that π can be written down in some numerical form, usually as a decimal. Some say it's about 3 while others might go on to say 3.14, or possibly something more precise. Of special note is Akira Haraguchi, a mental health counsellor from Japan, that can recite than 100,000 decimal digits of π from memory. However, no matter how precise you go, they're all just approximations.

One can find many exact expressions that equal π. For example

$$\begin{aligned}
\pi &= 2\left(\frac{2\times 2\times 4\times 4\times 6\times 6\times 8\times \dots}{1\times 3\times 3\times 5\times 5\times 7\times 7\times \dots}\right)\\
&= 4\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\dots\right)\\
&= \sqrt{\frac{6}{1^2}+\frac{6}{2^2}+\frac{6}{3^2}+\frac{6}{4^2}+\dots}\\
&= 3\left(1+\frac{1^2}{4\times 6}+\frac{1^2\times 3^2}{4\times 6\times 8\times 10}+\frac{1^2\times 3^2\times 5^2}{4\times 6\times 8\times 10\times 12\times 14}+\dots\right)
\end{aligned}$$

However, I think the most remarkable formula for the computation of π has got to be from India. Read on ...

1. Around 1910, the Indian mathematician Srinivasa Ramanujan discover the formula

$$\frac{1}{\pi}=\frac{2\sqrt{2}}{9801}\sum_{n=0}^{\infty}\frac{(4n)!\,(1103+26390n)}{(n!)^4\,396^{4n}}.$$

William Gosper (he's from New Jersey) used this series in 1985 to compute the first seventeen million digits of π.

(a) [5 points] Verify that this series is convergent.

(b) [5 points] How many correct decimal places of π do you get if you just use the first term of the series? What if you use two terms?