Name: _____

Signature: _____

Show all work clearly and in order, and box your final answers. Justify your answers whenever possible. You have 20 minutes to take this 10 point quiz.

1. [5 points] Given a differential equation of the form

$$y' = kxy^2,$$

find the constant k such that

$$y = \frac{1}{x^2+5}$$

is a solution to this differential equation.

Solution: First find y'

$$y' = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{x^2+5}\right] = -\frac{2x}{(x^2+5)^2},$$

and then see what k is in the differential equation,

$$\begin{aligned} y' &= kxy^2 \\ -\frac{2x}{(x^2+5)^2} &= kx\left(\frac{1}{x^2+5}\right)^2 \\ k &= \boxed{-2} \end{aligned}$$

2. [5 points] Consider the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x\left(1+y^2\right)}{2}, \quad y\left(0\right) = 1.$$

Sketch the solution to this initial value problem, and use your sketch to estimate $y\left(1\right)$. Also, given that

$$y\left(x\right) = \tan\left(\frac{x^2}{4} + \frac{\pi}{4}\right)$$

is a solution to this differential equation, estimate the true value of $y\left(1\right)$.

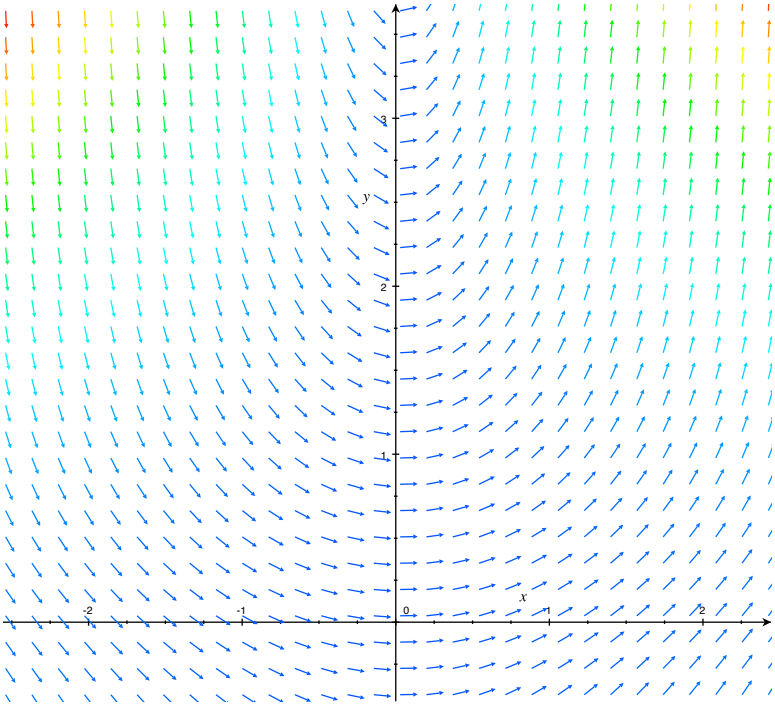

Figure 1: Direction field.

Solution: My sketch may be better than yours (I'm using software). From my graph I get $\boxed{y\left(1\right) = 1.7}$. Using the formula provided,

$$y\left(1\right) = \tan\left(\frac{1}{4} + \frac{\pi}{4}\right) \approx \boxed{1.68579641717}.$$

Not bad.