Name: ______________________________

Signature: ______________________________

Show all work clearly and in order, and box your final answers. Justify your answers whenever possible. You have 20 minutes to take this 10 point quiz.

1. Given

$$\int_0^5 \frac{4x}{\sqrt{x^4+1}}\ \mathrm{d}x,$$

and the following graph of this area.

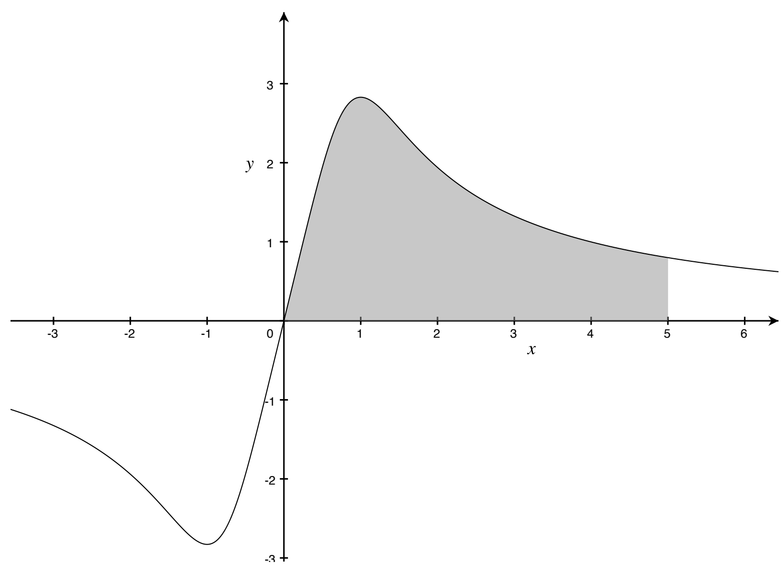

Figure 1: Area of interest.

Find the following.

(a) 9 points Evaluate (exact answer) the integral by initially using a simple u-substitution[1] followed by a trigonometric substitution.[2]

Solution: First let $u = x^2$, then $\mathrm{d}u = 2x\ \mathrm{d}x$.

$$\int_0^5 \frac{4x}{\sqrt{x^4+1}}\ \mathrm{d}x = 2\int_0^{25} \frac{1}{\sqrt{u^2+1}}\ \mathrm{d}u$$

[1] $u = x^2$
[2] $u = \tan\theta$

Now use $u = \tan\theta$, then $\mathrm{d}u = \sec^2\theta\ \mathrm{d}\theta$.

$$\begin{aligned} 2\int_0^{25} \frac{1}{\sqrt{u^2+1}}\ \mathrm{d}u &= 2\int_0^{\arctan 25} \frac{1}{\sqrt{\tan^2\theta+1}}\ \sec^2\theta\ \mathrm{d}\theta \\ &= 2\int_0^{\arctan 25} \sec\theta\ \mathrm{d}\theta \\ &= 2\ln|\sec\theta+\tan\theta|\ \Big]_0^{\arctan 25} = \boxed{2\ln\left(\sqrt{626}+25\right)} \end{aligned}$$

(b) 1 point For this integral, Mathematica returns $2\sinh^{-1} 25$. Is your answer equivalent?[3]

Solution: Yes, here's why. Recall[4] from the worksheet on hyperbolic functions that $\sinh^{-1} x = \ln\left(x+\sqrt{x^2+1}\right)$.

$$2\sinh^{-1} 25 = 2\ln\left(25+\sqrt{626}\right)$$

If you used your calculator you'd also see[5] that

$$2\sinh^{-1} 25 - 2\ln\left(25+\sqrt{626}\right) = 0.$$

[3] You may use a calculator.