Name: _____

Signature: _____

Show all work clearly and in order, and box your final answers. Justify your answers whenever possible. You have 20 minutes to take this 10 point quiz.

1. Show that the circumference of a unit circle is 2π.

Solution: I am using

$$x^2 + y^2 = 1$$

as the equation of a unit circle, where

$$y = \sqrt{1 - x^2}, \quad x \in [0,\ 1]$$

represents one-quarter of the circumference. Using the arc length formula I get

$$\begin{aligned} 4 \int_0^1 \frac{1}{\sqrt{1-x^2}}\, \mathrm{d}x &= 4 \lim_{a \to 1^-} \int_0^a \frac{1}{\sqrt{1-x^2}}\, \mathrm{d}x \\ &= 4 \lim_{a \to 1^-} \arcsin x \Big]_0^a \\ &= 4 \cdot \frac{\pi}{2} = 2\pi \end{aligned}$$

2. Show that the surface area of a unit sphere is 4π.

Solution: I am using

$$x^2 + y^2 = 1$$

as the equation of a unit circle, where

$$y = \sqrt{1 - x^2}, \quad x \in [0,\ 1]$$

represents one-quarter of the circumference. I will rotate this arc along the y-axis to get one-half of the surface area. Using the surface area formula I get

$$\begin{aligned} 2 \int_0^1 2\pi x \cdot \frac{1}{\sqrt{1-x^2}}\, \mathrm{d}x &= -2\pi \int_1^0 u^{-1/2}\, \mathrm{d}u \\ &= -2\pi \lim_{a \to 0^+} \int_1^a u^{-1/2}\, \mathrm{d}u \\ &= -2\pi \lim_{a \to 0^+} 2\sqrt{u} \Big]_1^a \\ &= -2\pi\,(-2) = 4\pi \end{aligned}$$