

Name: _____

Signature: _____

Show all work clearly and in order, and box your final answers. Justify your answers whenever possible. You have 20 minutes to take this 10 point quiz.

- The questions that follow relate to what is taught in MTH-120 at Essex County College, and what we will continue with in MTH-122. But more importantly me, it really relates to the type of questions that are typically used to determine a student's mathematical mettle. It is typical that at most competitive schools the exams don't necessarily reflect rote memorization, or routine problems. Here's just one example ...

We want to find the value of

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$$

and then make a conjecture about the value of

$$\sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \dots}}}}$$

for $x > 0$. The questions that follow should lead you in the direction of figuring this out.

- (a) 1 point Consider the recursive sequence

$$a_0 = 0, \quad a_{n+1} = \sqrt{1 + a_n}.$$

Compute the next five terms a_1 through a_5 .

- (b) 2 points Show that $a_n < 2$ for all n

(c) 2 points Show that $a_{n+1} > a_n$.

(d) 2 points Since $\{a_n\}$ is increasing and bounded above by 2, the **Monotone Sequence Theorem** says that $\{a_n\}$ converges, that is

$$\lim_{n \rightarrow \infty} a_n = a.$$

So if

$$\lim_{n \rightarrow \infty} a_n = a.$$

show that $a = \sqrt{1 + a}$.

(e) 1 point What is the value of

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$$

(f) 2 points Using similar reasoning, try to compute:

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}}$$