Essex County College
Division of Mathematics

Name: ______________________________

Signature: ______________________________

Show all work clearly and in order, and box your final answers. Justify your answers whenever possible. You have 20 minutes to take this 10 point quiz.

1. Consider the function

$$f(x) = \int_0^x te^{-t^3}\ \mathrm{d}t.$$

Unfortunately, it is not possible to write the formula for f any more explicitly than that. Doing so would involve computing a symbolic antiderivative of te^{-t^3}, which is impossible. However, we do not have to give up on working with such a function.

(a) 10 points [1] Show that near $x = 0$,

$$f(x) \approx \frac{x^2}{2} - \frac{x^5}{5} + \frac{x^8}{16} - \frac{x^{11}}{66}.$$

Solution: This could be obtained using Taylor's Formula for

$$te^{-t^3} = t - t^4 + \frac{t^7}{2!} - \frac{t^{10}}{3!} + \cdots,$$

and then integrating it to get

$$\begin{aligned} f(x) &= \int_0^x te^{-t^3}\ \mathrm{d}t \\ &= \int_0^x t - t^4 + \frac{t^7}{2!} - \frac{t^{10}}{3!} + \cdots\ \mathrm{d}t \\ &= \frac{x^2}{2} - \frac{x^5}{5} + \frac{x^8}{16} - \frac{x^{11}}{66} + \cdots. \end{aligned}$$

[1] In class part.

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(b) 10 points [2] One way to compare $f(x)$ and its approximation is to graph them both. Graphing $f(x)$ can be problematic, even with computer technology. There is another way to see how accurate our estimate is. On the same axes, graph

$$y = xe^{-x^3} \quad \text{and} \quad y = x - x^4 + \frac{x^7}{2} - \frac{x^{10}}{6}.$$

Are the graphs of these two functions similar near $x = 0$? Should we expect them to be?

Solution: The functions are similar, and they should be, because they are the graphs of $f'(x)$ and the derivative of the Taylor polynomial.

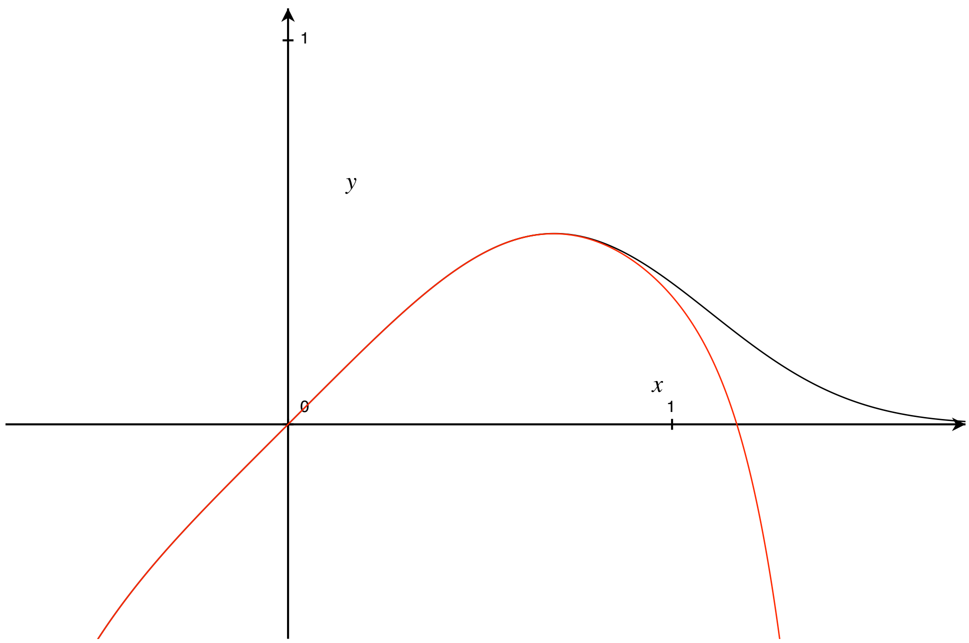

Figure 1: Partial graph of $f'(x)$ and $A'(x)$ [Red].

[2]Take-home part due next class!

Solution: I'm using Grapher (Mac OS X) and it's capable of graphing the actual integral. Please learn to use whatever technology you have at hand, it will aid you greatly in your studies.

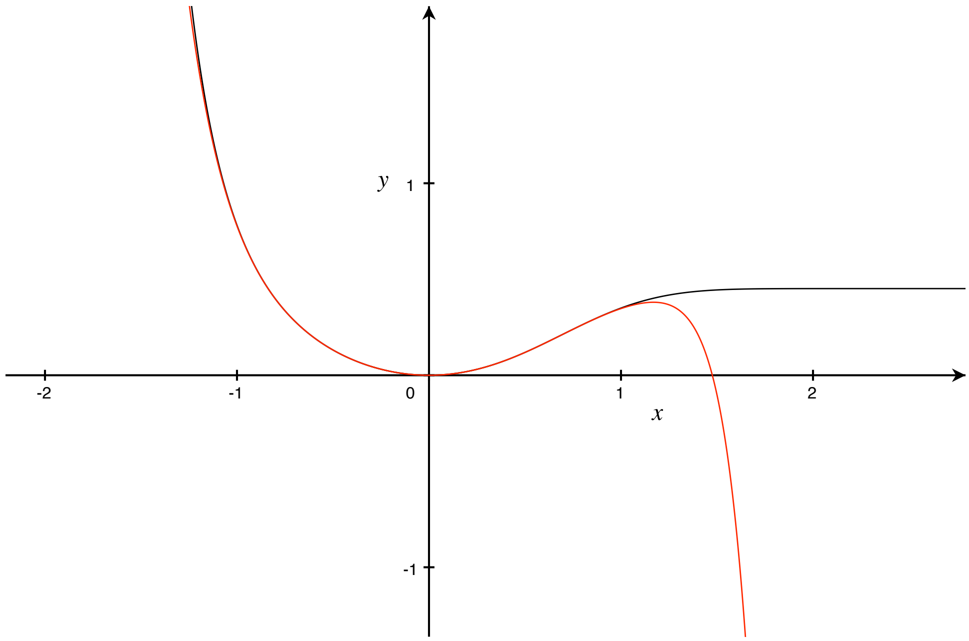

Figure 2: Partial graph of $f(x)$ and $A(x)$ [Red].