Name: ______________________________

Signature: ______________________________

Show all work clearly and in order, and box your final answers. Justify your answers whenever possible. You have 20 minutes to take this 10 point quiz.

Do one of the following three problems.

1. Find all values of x for which the following series converges.[1]

$$\sum_{n=1}^{\infty}\left(\frac{x}{n}-\frac{1}{n+1}\right)$$

Solution:

$$\begin{aligned} S_n &= \left(\frac{x}{1}-\frac{1}{2}\right)+\left(\frac{x}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{x}{n}-\frac{1}{n+1}\right) \\ &= x\sum_{i=1}^{n}\frac{1}{i}-\sum_{i=1}^{n}\frac{1}{i}+1-\frac{1}{n+1} \\ &= 1-\frac{1}{n+1}+(x-1)\sum_{i=1}^{n}\frac{1}{i} \end{aligned}$$

Since the harmonic diverges you want it to disappear from the partial sum, so just choose $x=1$ and you'll get

$$S_n = 1-\frac{1}{n+1}$$

So if $\boxed{x=1}$ we have

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)=1$$

2. Find all positive values of x for which the series converges.[2]

$$\sum_{n=1}^{\infty}x^{\ln n}$$

[1]**Hint:** Use partial sums, and try to find a value for x such that the harmonic series disappears.
[2]**Hint:** involves rewriting the series to look like a p-series.

Solution:

$$x^{\ln n} = \left(e^{\ln x}\right)^{\ln n} = \left(e^{\ln n}\right)^{\ln x} = n^{\ln x} = \frac{1}{n^{-\ln x}}$$

Now we see that it's a p-series, and it will converge for all x such that

$$\begin{aligned} -\ln x &> 1 \\ \ln x &< -1 \\ x &< e^{-1} \end{aligned}$$

So the series converges for $x < e^{-1}$.

3. The Riemann zeta-function[3] ζ is defined by

$$\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}$$

and is used in number theory to study the distribution of prime numbers. What is the domain of ζ?

Solution: It's a p-series, and it will converge for all x such that $x > 1$.

[3] http://mathworld.wolfram.com/RiemannZetaFunction.html