Name: _____

Signature: _____

Show all work clearly and in order, and box your final answers. Justify your answers whenever possible. You have 80 minutes to take this 100 point exam.

1. Consider the three infinite series below.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{5n}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)(n^2-1)}{4n^3-2n+1}$ (iii) $\displaystyle\sum_{n=1}^{\infty} \frac{5(-4)^{n+2}}{3^{2n+1}}$

(a) [10 points] Which of these series is (are) alternating?

(b) [10 points] Which one of these series diverges, and why?

(c) [10 points] One of these series converges absolutely. Which one? Compute its sum.

2. [10 points] For what values of p is the series convergent?

$$\sum_{n=2}^{\infty} (-1)^{n-1} \frac{(\ln n)^p}{n}$$

3. [10 points] Find the sum of

$$\sum_{n=1}^{\infty} \frac{k^{n-1}}{(n-1)!} e^{-k}.$$

4. [10 points] For which positive integers k is the following series convergent?

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$$

5. Using what you already know[1] about the Taylor series for e^x.

(a) [10 points] Find the Taylor series for

$$\cosh x = \frac{e^x + e^{-x}}{2}.$$

[1] Please do not take derivatives.

(b) [10 points] Looking at the Taylor series for $\cosh x$, explain why it looks like a parabola near $x = 0$. What is the equation of this parabola? Graph both $\cosh x$ and the parabola to see if it's a good fit near zero.

6. [10 points] By looking at the Taylor series, decide which of the folowing functions is largest, and which is smallest, for small positive θ.

$$1 + \sin\theta, \qquad \cos\theta, \qquad \frac{1}{1-\theta^2}$$

7. [10 points] Find the sum of

$$\sum_{n=1}^{\infty} nx^{n-1}$$

for $|x| < 1$.