1. Find the domain of

\[ f(x) = \frac{x^2 - 49}{\sqrt{x^2 + 9} - 5}. \]

**Solution:** \( x^2 + 9 \) is always positive so we don’t have to worry about the square root. However, to find the domain we need to solve \( \sqrt{x^2 + 9} - 5 = 0 \).

\[
\sqrt{x^2 + 9} = 5 \\
x^2 + 9 = 25 \\
x^2 = 16 \\
x = \pm 4
\]

So, the domain is \( \mathbb{R}, x \neq \pm 4 \).

2. Solve the inequality.

\[ |14 - x| - 3 < 17 \]

**Solution:**

\[
|14 - x| - 3 < 17 \\
|14 - x| < 20
\]

We know that for \( a > 0 \), that \( |x| < a \) is equivalent to \( -a < x < a \), so

\[
|14 - x| < 20 \\
-20 < 14 - x < 20 \\
-34 < -x < 6 \\
34 > x > -6 \\
-6 < x < 34
\]

Here the proper interval is: \( (-6, 34) \).

---

1This document was prepared by Ron Bannon using \LaTeXe.
3. Solve by using an augmented matrix and elementary row operations.

\[
\begin{align*}
2x + 3y - z &= -7 \\
3x - 3y + z &= 12 \\
2x + 4y + z &= -3
\end{align*}
\]

**Solution:** Augmented matrix form of the system:

\[
\begin{bmatrix}
2 & 3 & -1 & | & -7 \\
3 & -3 & 1 & | & 12 \\
2 & 4 & 1 & | & -3
\end{bmatrix}
\]

Elementary row operations, in order given:

\[R_1 + R_2 \rightarrow R_2\]
\[R_1 + R_3 \rightarrow R_3\]

Produces:

\[
\begin{bmatrix}
2 & 3 & -1 & | & -7 \\
3 & -3 & 1 & | & 5 \\
2 & 4 & 1 & | & 0
\end{bmatrix} \sim \begin{bmatrix}
2 & 3 & -1 & | & -7 \\
4 & 7 & 0 & | & -10
\end{bmatrix}
\]

The second row gives \(x = 1\); using \(x = 1\) in row three, gives \(y = -2\); finally, using \(x = 1\) and \(y = -2\) in row one, gives \(z = 3\).

4. Use long division to find the quotient and remainder when \(x^4 - 4x^2 + 2x + 5\) is divided by \(x - 2\).

**Solution:** Doing the division gives a remainder of 9 and a quotient of \(x^3 + 2x^2 + 2\).

5. Is \(x = -1\) a root of the polynomial function \(f(x) = 2x^3 - 5x^2 - 4x + 3\)?

**Solution:** Yes, because \(f(-1) = 0\).

6. Factor \(^2\)

\(f(x) = 2x^3 - 5x^2 - 4x + 3\).

**Solution:** Using the fact that \(x = -1\) is a root, we can divide \(f(x)\) by \(x + 1\), getting

\[
\frac{2x^3 - 5x^2 - 4x + 3}{x + 1} = 2x^2 - 7x + 3 = (2x - 1)(x - 3),
\]

so the complete factorization of \(f(x)\) is

\[(x + 1)(2x - 1)(x - 3)\].

\(^2\)Previous problem may be helpful.
7. Find the area of the triangle with vertices \((-1, 0), (2, 1)\) and \((1, -2)\).

Solution:

\[
A = \pm \frac{1}{2} \begin{vmatrix} -1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = \pm \frac{1}{2} [-1(3) - 0(1) + 1(-5)] = \pm 4
\]

The area is 4 square units.

8. Solve each inequality, and use interval notation to express the solution.

(a) \(\frac{1}{x} \geq \frac{1}{x^3}\)

Solution:

\[
\frac{1}{x} \geq \frac{1}{x^3} \\
\frac{1}{x} - \frac{1}{x^3} \geq 0 \\
\frac{x^2 - 1}{x^3} \geq 0 \\
\frac{(x - 1)(x + 1)}{x^3} \geq 0
\]

Using a number line, you’ll get \([-1, 0) \cup [1, \infty)\].

(b) \((1 - x)^2 \leq 10 - 2x\)

Solution:

\[
(1 - x)^2 \leq 10 - 2x \\
1 - 2x + x^2 \leq 10 - 2x \\
x^2 - 9 \leq 0 \\
(x - 3)(x + 3) \leq 0
\]

Using a number line, you’ll get \([-3, 3]\).

9. Solve for \(x\).

(a) \(\log_5 x = 2\)

Solution:

\[
\log_5 x = 2 \iff x = 5^2 \implies x = 25
\]
(b) \( \log_x 64 = 3 \)

**Solution:**
\[
\log_x 64 = 3 \quad \iff \quad x^3 = 64 \quad \Rightarrow \quad x = 4
\]

(c) \( \log_3 \frac{1}{9} = x \)

**Solution:**
\[
\log_3 \frac{1}{9} = x \quad \iff \quad 3^x = \frac{1}{9} \quad \Rightarrow \quad x = -2
\]

10. Factor

\[
f(x) = (x^2 + x + 1) \left(6x^2 + 5x - 6\right)
\]

into linear factors.

**Solution:** You’ll need to use the quadratic formula on \( x^2 + x + 1 = 0 \), which gives:

\[
x = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2},
\]

so \( x^2 + x + 1 \) factors into two linear factors, as follows

\[
x^2 + x + 1 = \left(x - \frac{-1 + \sqrt{3}i}{2}\right)\left(x - \frac{-1 - \sqrt{3}i}{2}\right)
\]

\[
= \frac{1}{4} \left(2x + 1 - \sqrt{3}i\right)\left(2x + 1 + \sqrt{3}i\right).
\]

You can also use the quadratic formula to factor \( 6x^2 + 5x - 6 \), but I think it is easier to use intelligent trial and error, as follows

\[
6x^2 + 5x - 6 = (2x + 3) \left(3x - 2\right).
\]

Finally, as requested, the complete factorization is

\[
f(x) = \frac{1}{4} \left(2x + 3\right) \left(3x - 2\right) \left(2x + 1 - \sqrt{3}i\right) \left(2x + 1 + \sqrt{3}i\right)
\]

11. A total of $17,500 is invested at an annual rate of 9.3\%, compounded weekly. Find the balance after 35 years.

\[
17500 \cdot \left(1 + \frac{0.093}{52}\right)^{(52 \cdot 35)} \approx 452276.46
\]

So, the balance is $452,276.46.
12. Solve the inequality.
\[ \frac{3 - 2x}{x - 1} + 2 \geq 0 \]

Solution:
\[ \frac{3 - 2x}{x - 1} + 2 \geq 0 \]
\[ 3 - 2x + 2(x - 1) \geq 0 \]
\[ 3 - 2x + 2x - 2 \geq 0 \]
\[ \frac{1}{x - 1} \geq 0 \]

Using simple sign analysis, the proper interval is: \((1, \infty)\).

13. Solve for \(x\).
\[ \log_3 (2x + 1) + \log_3 (2x - 1) = 1 \]

Solution:
\[ \log_3 (2x + 1) + \log_3 (2x - 1) = 1 \]
\[ \log_3 (2x + 1) (2x - 1) = 1 \]
\[ \log_3 (4x^2 - 1) = 1 \]
\[ 4x^2 - 1 = 3 \]
\[ 4x^2 - 4 = 0 \]
\[ x^2 - 1 = 0 \]
\[ x = \pm 1 \]

The solution is \(x = 1\) and you should check this. I will also give full credit for \(x = \pm 1\).

14. Find the product.
\[ \begin{bmatrix} 3 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 2 \\ -1 & -5 & -1 \\ 0 & 1 & -1 \end{bmatrix} \]

Solution:
\[ \begin{bmatrix} 3 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 2 \\ -1 & -5 & -1 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 5 \\ 2 & 19 & 5 \end{bmatrix} \]

If you’re limited to using real numbers only, whereas complex numbers can look mighty strange, such as the famous—or infamous—identity \(e^{i\pi} + 1 = 0\).
15. Given 

\[ f(x) = \frac{2x - 1}{2 - 3x}, \]

find and simplify the difference quotient

\[ \frac{f(x + h) - f(x)}{h}, \quad h \neq 0. \]

**Solution:** Do I need to say it? Yes, the following is true if and only if \( h \neq 0. \)

\[
\begin{align*}
\frac{f(x + h) - f(x)}{h} &= \frac{2x + 2h - 1}{2 - 3x - 3h} + \frac{2x - 1}{2 - 3x} \\
&= \frac{(2x + 2h - 1)(2 - 3x) - (2x - 1)(2 - 3x - 3h)}{h(2 - 3x - 3h)(2 - 3x)} \\
&= \frac{4x + 4h - 2 - 6x^2 - 6xh + 3x - 4x + 2 + 6x^2 - 3x + 6xh - 3h}{h(2 - 3x - 3h)(2 - 3x)} \\
&= \frac{h}{h(2 - 3x - 3h)(2 - 3x)} \\
&= \frac{1}{(2 - 3x - 3h)(2 - 3x)}
\end{align*}
\]

16. Find the inverse, if possible.

\[
\begin{bmatrix}
1 & 1 & 1 \\
3 & 5 & 4 \\
3 & 6 & 5
\end{bmatrix}
\]

**Solution:**

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 \\
3 & 5 & 4 & 0 & 1 & 0 \\
3 & 6 & 5 & 0 & 0 & 1
\end{bmatrix} \sim \begin{bmatrix}
1 & 0 & 0 & 1 & 1 & -1 \\
0 & 1 & 0 & -3 & 2 & -1 \\
0 & 0 & 1 & 3 & -3 & 2
\end{bmatrix}
\]

So the inverse is

\[
\begin{bmatrix}
1 & 1 & -1 \\
-3 & 2 & -1 \\
3 & -3 & 2
\end{bmatrix}
\]

17. Solve the system, any method is fine.

\[
\begin{align*}
x + y - z &= 4 \\
-3x + 2y - z &= -6 \\
3x - 3y + 2z &= 5
\end{align*}
\]
**Solution**: You may have noticed that you found the inverse of the coefficient matrix in the prior problem. So I’ll solve using the inverse.

\[
\begin{bmatrix}
1 & 1 & -1 \\
-3 & 2 & -1 \\
3 & -3 & 2
\end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 5 \end{bmatrix}
\]

\[
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\
3 & 5 & 4 \\
3 & 6 & 5 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -6 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}
\]

I get \( x = 3 \), \( y = 2 \) and \( z = 1 \) which can be easily checked.

18. Identify the elementary row operation performed on the original matrix (left) to obtained the new row-equivalent matrix (right).

\[
\begin{bmatrix}
-2 & 5 & 1 \\
3 & -1 & -8
\end{bmatrix} \sim \begin{bmatrix}
13 & 0 & -39 \\
3 & -1 & -8
\end{bmatrix}
\]

**Solution**: By inspection:

\( 5R_2 + R_1 \rightarrow R_1 \).

19. Find the domain of

\( f(x) = \frac{x + 2}{\sqrt{2x^2 - 18}} \).

**Solution**: Finding the domain requires solving \( 2x^2 - 18 > 0 \) for \( x \).

\[
2x^2 - 18 > 0 \quad \Rightarrow \quad x^2 - 9 > 0 \quad \Rightarrow \quad (x - 3)(x + 3) > 0
\]

Simple sign analysis gives \((-\infty, -3) \cup (3, \infty)\).

20. Given

\( f(x) = (x + 3)^2, \quad x \geq -3 \),

find \( f^{-1}(x) \). Graphing may be helpful, but is not required.

**Solution**: The domain of \( f(x) \) is \([-3, \infty)\) and the range is \([0, \infty)\), so the domain of \( f^{-1}(x) \) is \([0, \infty)\) and the range is \([-3, \infty)\).

\[
f(x) = (x + 3)^2 \\
y = (x + 3)^2 \\
x = (y + 3)^2 \\
\pm \sqrt{x} = y + 3 \\
\pm \sqrt{x} - 3 = y
\]
However, since \( y \geq -3 \) we have

\[
    f^{-1}(x) = \sqrt{x} - 3, \quad x \geq 0.
\]

21. Given

\[
    f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18, \quad \text{and} \quad f(2) = f(-3) = 0
\]

answer the following questions.

(a) Use the given roots, and long division, to completely factor \( f(x) \).

\textbf{Solution:} Since we are given two roots, we know two factors of \( f(x) \) are \((x - 2)\) and \((x + 3)\). Dividing \( f(x) \) by the product of these two factors gives

\[
    2x^2 + 5x + 3 = (2x + 3)(x + 1).
\]

So the complete factorization of \( f(x) \) is

\[
    (2x + 3)(x + 1)(x - 2)(x + 3)
\]

(b) Graph \( f(x) \) using the roots, \( y \)-intercept and sign-analyses.

\textbf{Solution:} Your graph does not need to have such precise detail as mine, but it should still reflect the key pre-calculus analysis.

![Figure 1: Graph of \( f(x) \).](image-url)
22. Given
\[ f(x) = \sqrt{x + 6} \quad \text{and} \quad g(x) = x^2 - 5, \]
answer each of the following questions.
(a) Find \((f \circ g)(x)\)

Solution:
\[ (f \circ g)(x) = f(g(x)) = f(x^2 - 5) = \sqrt{x^2 - 5 + 6} = \sqrt{x^2 + 1} \]

(b) The domain of \((f \circ g)(x)\)

Solution: \(\mathbb{R}\).

(c) Find \((g \circ f)(x)\)

Solution:
\[ (g \circ f)(x) = g(f(x)) = g(\sqrt{x + 6}) = (\sqrt{x + 6})^2 - 5 = x + 6 - 5 = x + 1 \]

(d) The domain of \((g \circ f)(x)\)

Solution: \(x \geq -6\).

23. Given
\[ f(x) = \frac{x^3 + 2x^2}{x^2 + 1}, \]
answer the following questions.
(a) \(x\)-intercept(s) in point form.

Solution: Set \(f(x) = 0\) and solve for \(x\).
\[ 0 = \frac{x^3 + 2x^2}{x^2 + 1} \]
\[ 0 = \frac{x^2(x + 2)}{x^2 + 1} \]

Clearly, \((-2, 0)\) and \((0, 0)\).

(b) \(y\)-intercept in point form.

Solution: Set \(x = 0\) and evaluate. Clearly, \((0, 0)\).

(c) All linear asymptotes in equation form.

Solution: Since the degree of the numerator is one more than the denominator, we have the possibility of getting a slant asymptote. The long division gives
\[ f(x) = \frac{x^3 + 2x^2}{x^2 + 1} = x + 2 - \frac{x + 2}{x^2 + 1}, \]
so the slant asymptote is \( y = x + 2 \).

(d) Graph \( f(x) \) using the information above and sign-analyses.

**Solution**: Your graph does not need to have such precise detail as mine, but it should still reflect the key pre-calculus analysis.

![Graph of \( f(x) \).](image)

Figure 2: Graph of \( f(x) \).

24. Given

\[
 f(x) = x^2 - x + 1,
\]

find and simplify the difference quotient

\[
 \frac{f(x+h) - f(x)}{h}, \quad h \neq 0.
\]

**Solution**: Do I need to say it? Yes, the following is true if and only if \( h \neq 0 \).

\[
 \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - (x+h) + 1 - (x^2 - x + 1)}{h}
 = \frac{x^2 + 2xh + h^2 - x - h + 1 - x^2 + x - 1}{h}
 = \frac{2xh + h^2 - h}{h}
 = 2x + h - 1
\]
25. Given the following functional graph. Answer the following questions.

(a) Is \( f(x) \) invertible? Explain your answer.

**Solution:** No. It fails the horizontal line test.

(b) Graph

\[ 2 \cdot f \left( 1 - \frac{x}{2} \right) - 1 \]

**Solution:** The points translated, in order are

\[ \{(10, 3), (6, 5), (2, -3), (-6, -5)\} \].

Figure 3: Graph of \( f(x) \).

Figure 4: Graph of \( 2 \cdot f \left( 1 - \frac{x}{2} \right) - 1 \).
26. Solve the inequality. \[ |14 - 3x| - 5 > -2 \]

Solution:

\[
|14 - 3x| - 5 > -2 \\
|14 - 3x| > 3
\]

We know that \( |x| > a \) is equivalent to \( x > a \) or \( x < -a \), so

\[
|14 - 3x| > 3 \Rightarrow 14 - 3x > 3 \quad \text{or} \quad 14 - 3x < -3 \\
x < 11/3 \quad \text{or} \quad 17/3 < x
\]

Here the proper intervals are: \((-\infty, 11/3) \cup (17/3, \infty)\).

27. Sketch the graph of the function. \( f(x) = 3^{x+2} \)

Solution:

![Graph of \( f(x) \).](image)
28. Write the system of linear equations represented by the augmented matrix. Then use back-substitution to find the solution. (Use the variables $x$, $y$, and $z$.)

\[
\begin{bmatrix}
1 & -1 & 2 & 4 \\
0 & 1 & -1 & 2 \\
0 & 0 & 1 & -2
\end{bmatrix}
\]

Solution:

\[
\begin{aligned}
x - y + 2z &= 4 \\
y - z &= 2 \\
z &= -2
\end{aligned}
\]

The last line gives $z = -2$, then using the value for $z$ in line two, we get $y = 0$, finally using these two values in line one we get $x = 8$.

29. Condense the expression to the logarithm of a single quantity.

\[
\frac{1}{3} \left[ 2 \ln (x + 3) + \ln x - \ln (x^2 - 1) \right]
\]

Solution:

\[
\frac{1}{3} \left[ 2 \ln (x + 3) + \ln x - \ln (x^2 - 1) \right] = \frac{1}{3} \left[ \ln (x + 3)^2 + \ln x - \ln (x^2 - 1) \right] = \frac{1}{3} \left[ \ln \frac{x(x+3)^2}{x^2-1} \right] = \ln \sqrt[3]{\frac{x(x+3)^2}{x^2-1}}
\]

30. Evaluate the expression, if possible.

\[
\begin{bmatrix}
-1 & 6 \\
-4 & 5 \\
0 & 3
\end{bmatrix}
\begin{bmatrix}
2 & 3 \\
0 & 9
\end{bmatrix}
\]

Solution:

\[
\begin{bmatrix}
-1 & 6 \\
-4 & 5 \\
0 & 3
\end{bmatrix}
\begin{bmatrix}
2 & 3 \\
0 & 9
\end{bmatrix} = \begin{bmatrix}
-2 & 51 \\
-8 & 33 \\
0 & 27
\end{bmatrix}
\]
31. Solve for $x$.

\[
\left(\frac{2}{3}\right)^x = \frac{81}{16}
\]

Solution:

\[
\left(\frac{2}{3}\right)^x = \frac{81}{16}
\]

\[
\left(\frac{2}{3}\right)^x = \left(\frac{3}{2}\right)^4
\]

\[
\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^{-4}
\]

So, $x = -4$.

32. Solve the inequality.

\[
\frac{x + 6}{x + 1} - 2 < 0
\]

Solution:

\[
\frac{x + 6}{x + 1} - 2 < 0
\]

\[
\frac{x + 6}{x + 1} - \frac{2(x + 1)}{x + 1} < 0
\]

\[
\frac{4 - x}{x + 1} < 0
\]

Using simple sign analysis, the proper intervals are: $(-\infty, -1) \cup (4, \infty)$.

33. Solve algebraically (exact answer) and then approximate to three decimal places.

\[-2 + 2 \ln 3x = 17\]

Solution:

\[-2 + 2 \ln 3x = 17\]

\[2 \ln 3x = 19\]

\[\ln 3x = \frac{19}{2}\]

\[3x = e^{19/2}\]

\[x = \frac{e^{19/2}}{3} \approx 4,453.242\]
34. Find the determinant of the matrix.

\[
\begin{vmatrix}
-1 & 2 & -5 \\
0 & 3 & 4 \\
0 & 0 & 3 \\
\end{vmatrix}
\]

**Solution:**

\((-1) \cdot (9 - 0) - (2) \cdot (0 - 0) + (-5) \cdot (0 - 0) = -9\)

You might also noticed that the determinant, in this case, is just the product of the diagonal entries.

35. Given

\[f(x) = \frac{x^2 - 3x + 2}{2x^2 + 5x + 3} = \frac{(x - 1)(x - 2)}{(2x + 3)(x + 1)},\]

answer the following questions:

(a) x-intercepts in point form.

**Solution:** \((1, 0); (2, 0)\)

(b) y-intercept in point form.

**Solution:** \((0, \frac{2}{3})\)

(c) Equation of the horizontal asymptote.

**Solution:** \(y = \frac{1}{2}\)

(d) Equation of the vertical asymptotes.

**Solution:** \(x = -1; x = -\frac{3}{2}\)

(e) Graph \(f(x)\) using the above information and sign analysis.

![Graph of f(x)](image)

**Figure 6:** Graph of \(f(x)\).